Mechanical Engineering Department
University of New Mexico
Ph.D. Qualifying Examination
Controls Section
Fall 2018

INSTRUCTIONS:

- Time allowed: 2 hours
- Closed book and closed notes; one sheet (8.50 $\times$ 11.00 in, 2-sided) of formulas is allowed
- Laplace transform tables are included on the last two pages
- There are 4 problems worth a total of 100 points
- You MUST show and explain work to get credit
- Calculator allowed
- Laptops, cell phones, and similar electronic devices are not allowed

Problem 1 (25 points).

Consider the system described by the characteristic equation:

$$1 + \frac{K}{s^3 + 6s^2 + 5s} = 0$$

a) Sketch the root locus as K varies from $K = 0$ to $K \to \infty$.

b) Show all important calculations.

c) Clearly identify the real-axis segments (if any), the asymptotes (if any), and all break-away and/or break-in points.

d) For what value(s) of K are the roots on the imaginary axis?

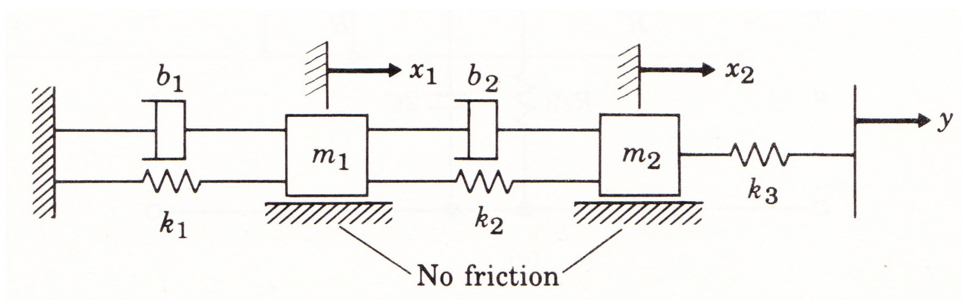
Problem 2 (25 points).

a) Derive the second-order differential equations for the positions x_1, x_2 in the mechanical system shown below.

Assume that the springs k_1, k_2, k_3 are linear springs, with $x_1 = 0, x_2 = 0, y = 0$ corresponding to the unstretched states of the springs.

Assume that the dampers b_1, b_2 are linear dampers.

Assume that y is a specified function of time.



b) Rewrite the second-order differential equations as a state-space system in the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

Problem 3 (25 points). A given system has transfer function

$$H(s) = \frac{0.5s}{s^4 + 2.2s^3 + 3s^2 + 1.1s + 1}$$

Where needed, use initial conditions $y(0) = 1,\ \dot{y}(0) = -1,\ \ddot{y}(0) = 2,\ \dddot{y}(0) = -2$.

a) Use Laplace transform methods to obtain the complete solution when the input is $u(t) = \sin 3t$.

b) Use Laplace transform methods to obtain the complete solution when the input is $u(t) = \delta(t)$; i.e., find the impulse response.

c) Construct a Bode plot for the system for input $u(t) = \sin \omega t$.

Problem 4 (25 points). A system has the following state space representation:

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u(t), \quad y = [1 \;\; 2 \;\; 3]\mathbf{x}$$

a) Derive the corresponding transfer function.

b) What are the system poles?

Laplace Transform Tables

TABLE A.1
Properties of Laplace Transforms

Number	Laplace transform	Time function	Comment
—	$F(s)$	$f(t)$	Transform pair
1	$\alpha F_1(s) + \beta F_2(s)$	$\alpha f_1(t) + \beta f_2(t)$	Superposition
2	$F(s)e^{-s\lambda}$	$f(t-\lambda)$	Time delay ($\lambda \geq 0$)
3	$\frac{1}{\lvert a\rvert}F\left(\frac{s}{a}\right)$	$f(at)$	Time scaling
4	$F(s+a)$	$e^{-at}f(t)$	Shift in frequency
5	$s^m F(s) - s^{m-1}f(0) - s^{m-2}\overset{\circ}{f}(0) - \cdots - f^{(m-1)}(0)$	$f^{(m)}(t)$	Differentiation
6	$\frac{1}{s}F(s) + \frac{[\overset{\circ}{\int f}(t)dt\|0}{s}$	$\int f(\zeta)d\zeta$	Integration
7	$F_1(s)F_2(s)$	$f_1(t) * f_2(t)$	Convolution
8	$\lim_{s\to\infty} sF(s)$	$f(0+)$	Initial-value theorem
9	$\lim_{s\to 0} sF(s)$	$\lim_{t\to\infty} f(t)$	Final-value theorem
10	$\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F_1(\zeta)F_2(s-\zeta)d\zeta$	$f_1(t)f_2f(t)$	Time product
11	$-\frac{d}{ds}F(s)$	$tf(t)$	Multiplication by time

TABLE A.2
Table of Laplace Transforms

Number	$F(s)$	$f(t), \quad t \geq 0$
1	1	$\delta(t)$
2	$1/s$	$1(t)$
3	$1/s^2$	t
4	$2!/s^3$	t^2
5	$3!/s^4$	t^3
6	$m!/s^{m+1}$	t^m
7	$1/(s+a)$	e^{-at}
8	$1/(s+a)^2$	te^{-at}
9	$1/(s+a)^3$	$\frac{1}{2!}t^2e^{-at}$
10	$1/(s+a)^m$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at-1+e^{-at})$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bt}-ae^{-at}$
17	$a/(s^2+a^2)$	$\sin at$
18	$s/(s^2+a^2)$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin bt$
21	$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$