Mechanical Engineering Department

University of New Mexico

Ph.D. Qualifying Examination

Controls Section

Fall

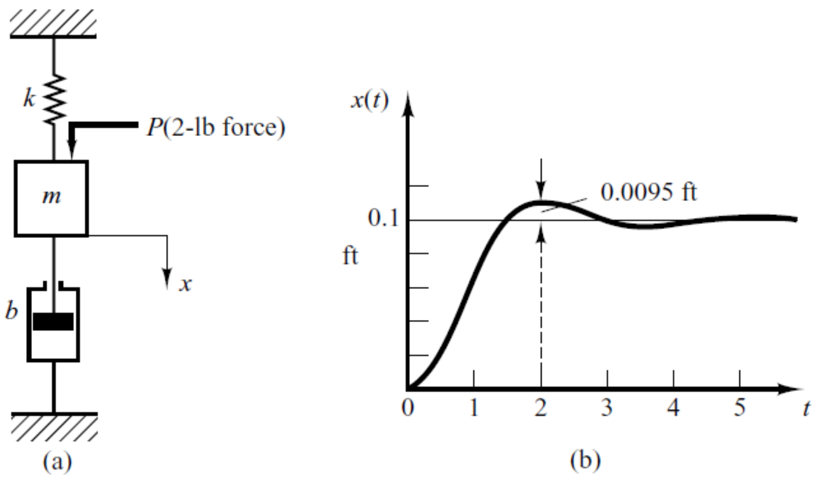
2019

INSTRUCTIONS:

- Closed Book
- There are 4 problems worth a total of 100 points
- 2 Hours
- Calculator allowed
- You ***MUST*** show work to get credit.

Problem 1. (25 points)

The figure shows a mechanical vibratory system. When 2 lb of force (step input) is applied to the system, the mass oscillates, as shown in the figure on the right. Determine *m*, *b*, and *k* of the system from this response curve. The displacement *x* is measured from the equilibrium position.



The formula for the peak time, the time at which *x(t)* reaches its maximum value is equal to

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1 - \zeta^2}}$$

Problem 2. (25 points)

A controlled process is modeled by the following state equations.

$$\frac{dx_1(t)}{dt} = x_1(t) - 2x_2(t) \quad \frac{dx_2(t)}{dt} = 10x_1(t) + u(t)$$

The control $u(t)$ is obtained from state feedback such that

$$u(t) = -k_1 x_1(t) - k_2 x_2(t)$$

where k_1 and k_2 are real constants. Determine the region in the k_1-versus-k_2 parameter plane in which the closed-loop system is asymptotically stable.

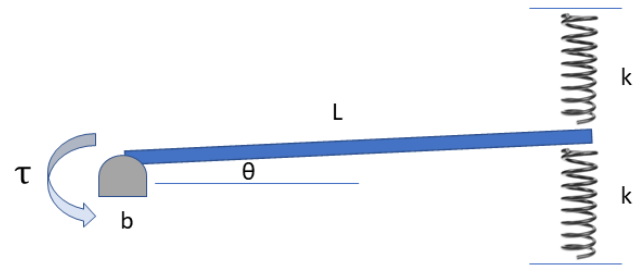
Problem 3 (25 points)

For the following transfer function, $\mathrm{G(s)} = \frac{s+1}{s^2(s+4)}$

a) Sketch the root locus for $1 + \mathrm{KG(s)} = 0$

b) Determine the value of K at the pole at the far left of the complex plane.

Problem 4 (25 points)



The dynamic equation follows the below formula.

$$J\ddot{\theta} + b\dot{\theta} + 2kL^2 sin\theta = \tau$$

Assume *J=2, b=6,* kL^2*=2* and small changes in the angle.

a) Derive the Transfer Function
b) For a unit step input, find the solution of θ as a function of time, by setting the initial conditions equal zero.