# Mechanical Engineering Department
# University of New Mexico

# Ph.D. Qualifying Examination

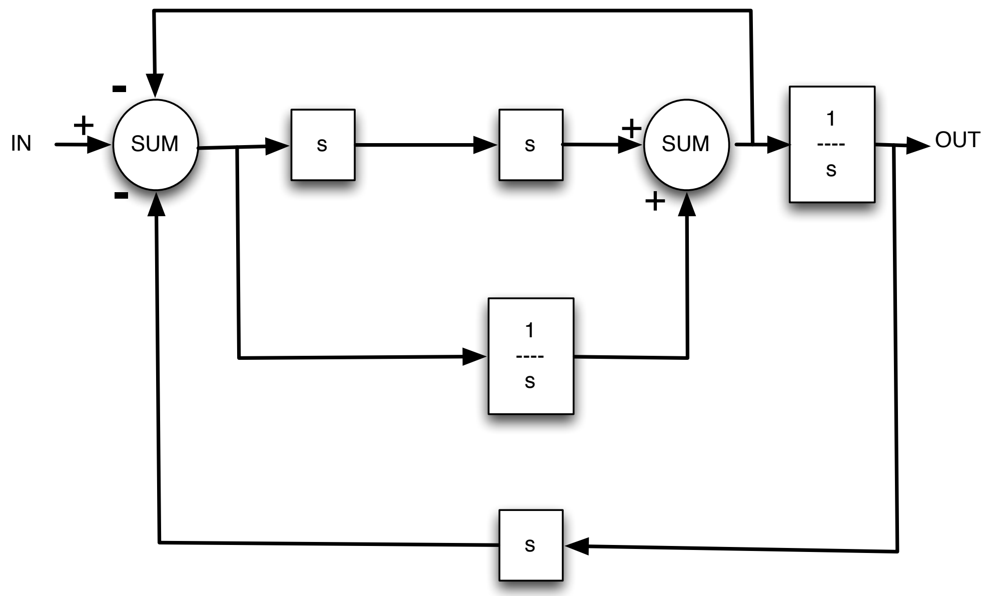
# Controls Section

# Fall
# 2016

**INSTRUCTIONS**:

- Closed Book
- There are 3 problems worth a total of 100 points
- 2 Hours
- Calculator allowed
- You ***MUST*** show work to get credit.

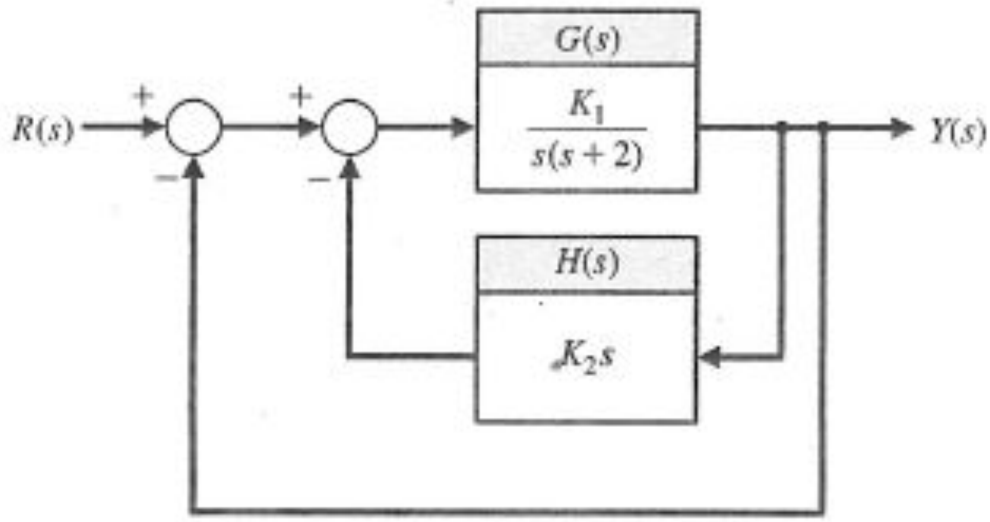
**Problem 1. (30 points)**




For the system above, find an equivalent system that has no feedback paths. Show work.

**Problem 2. (40 points)**

Consider the feedback system in the figure, representing an accurate control system for positioning a welding head:




Use the root locus to select the amplifier gain $K_1$ and the derivative feedback gain $K_2$ so that the following specifications are satisfied:

1) Steady state error for a unit ramp $e_{ss} \leq 0.35$
2) Damping ratio of dominant root $\zeta \geq 0.707$
3) Settling time to within 2% of the final value is equal to $T_s \leq 3$s

**Problem 3. (30 points)**

For the system shown below, find the system type and the steady state error for a unit step input. Show work.

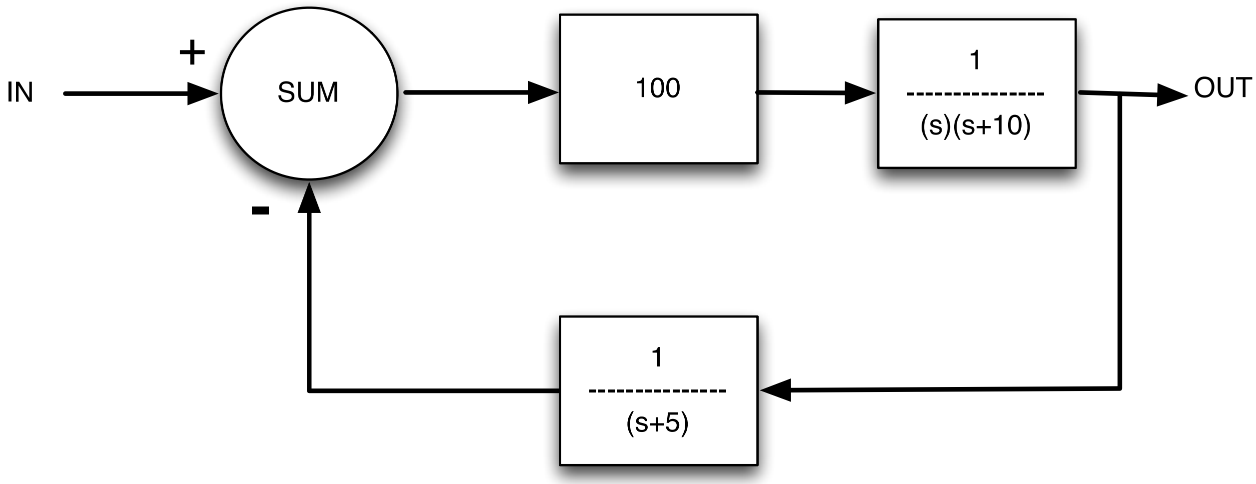


**Table 2.1** Laplace transform table

| Item no. | $f(t)$ | $F(s)$ |
|---|---|---|
| 1. | $\delta(t)$ | $1$ |
| 2. | $u(t)$ | $\frac{1}{s}$ |
| 3. | $tu(t)$ | $\frac{1}{s^2}$ |
| 4. | $t^n u(t)$ | $\frac{n!}{s^{n+1}}$ |
| 5. | $e^{-at}u(t)$ | $\frac{1}{s+a}$ |
| 6. | $\sin \omega t u(t)$ | $\frac{\omega}{s^2+\omega^2}$ |
| 7. | $\cos \omega t u(t)$ | $\frac{s}{s^2+\omega^2}$ |

**Table 2.2** Laplace transform theorems

| Item no. | Theorem | Name |
|---|---|---|
| 1. | $\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$ | Definition |
| 2. | $\mathscr{L}[kf(t)] = kF(s)$ | Linearity theorem |
| 3. | $\mathscr{L}[f_1(t) + f_2(t)] = F_1(s) + F_2(s)$ | Linearity theorem |
| 4. | $\mathscr{L}[e^{-at}f(t)] = F(s+a)$ | Frequency shift theorem |
| 5. | $\mathscr{L}[f(t-T)] = e^{-sT}F(s)$ | Time shift theorem |
| 6. | $\mathscr{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$ | Scaling theorem |
| 7. | $\mathscr{L}\left[\frac{df}{dt}\right] = sF(s) - f(0-)$ | Differentiation theorem |
| 8. | $\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0-) - \dot{f}(0-)$ | Differentiation theorem |
| 9. | $\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$ | Differentiation theorem |
| 10. | $\mathscr{L}\left[\int_{0-}^{t} f(\tau)\,d\tau\right] = \frac{F(s)}{s}$ | Integration theorem |
| 11. | $f(\infty) = \lim_{s \to 0} sF(s)$ | Final value theorem[1] |
| 12. | $f(0+) = \lim_{s \to \infty} sF(s)$ | Initial value theorem[2] |

[1] For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts and no more than one can be at the origin.
[2] For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (i.e., no impulses or their derivatives at $t = 0$).