Mechanical Engineering Department
University of New Mexico

Ph.D. Qualifying Examination

Controls Section

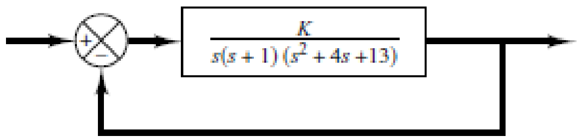
Spring
2016

INSTRUCTIONS:

- Closed Book
- There are 4 problems worth a total of 100 points
- 2 Hours
- Calculator allowed
- You ***MUST*** show work to get credit.

Problem 1. (25 points)

Sketch the root locus for the system in the figure:



Explain which rules you applied.

Table 2.1 Laplace transform table

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

Table 2.2 Laplace transform theorems

Item no.	Theorem	Name
1.	$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)] = kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t) + f_2(t)] = F_1(s) + F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)] = F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)] = e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0-) - \dot{f}(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)\,d\tau\right] = \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) = \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+) = \lim_{s\to \infty} sF(s)$	Initial value theorem[2]

[1] For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts and no more than one can be at the origin.

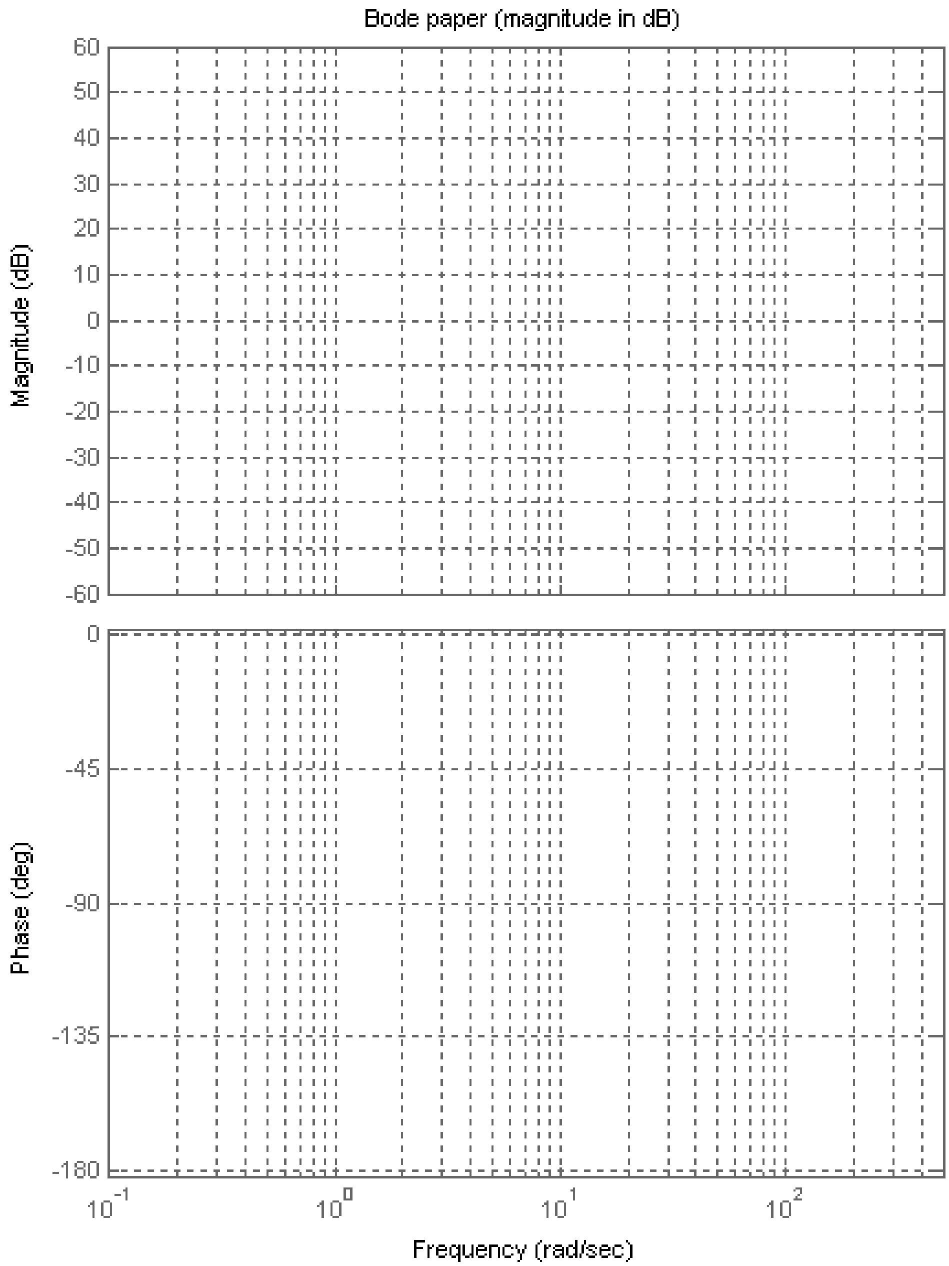
[2] For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (i.e., no impulses or their derivatives at $t = 0$).

Problem 2. (25 points)

Sketch the asymptotic Bode plots for the transfer function

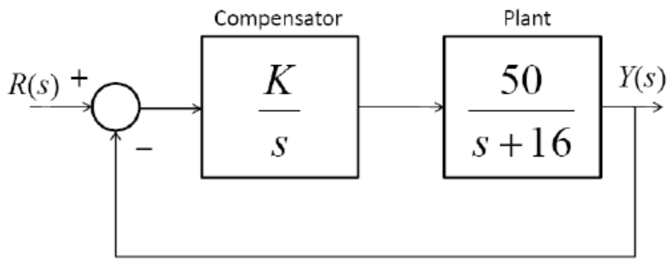
$$G(s) = \frac{10(s+3)}{s(s+0.2)(s^2+15s+100)}$$

Use the graph paper on the following page.


Bode paper (magnitude in dB)
Magnitude (dB)
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
-60
Phase (deg)
0
-45
-90
-135
-180
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)

Problem 3 (25 points)

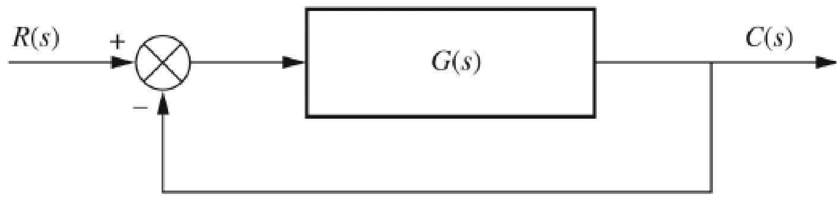
For the unity feedback system shown below,
(a) specify the gain K of the compensator so that the overall closed-loop response to a unit-step input has a damping ratio of 0.517,
(b) for the value of K in (a) compute the peak-time, T_p, and damped frequency, ω_d.



Problem 4 (25 points)

Using the Routh table determine the values of K > 0 and a > 0 so that the unity feedback system shown oscillates at a frequency 3 rad/s, when

$$G(s) = \frac{K(s+1)}{s^3 + as^2 + 3s + 1}$$



Name	Time function, $f(t)$	Laplace transform, $F(s)$
Unit impulse	$\delta(t)$	1
Unit step	$u(t)$	$\frac{1}{s}$
Unit ramp	t	$\frac{1}{s^2}$
nth-order ramp	t^n	$\frac{n!}{s^{n+1}}$
Exponential	e^{-at}	$\frac{1}{s+a}$
nth-order exponential	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
Sine	$\sin bt$	$\frac{b}{s^2+b^2}$
Cosine	$\cos bt$	$\frac{s}{s^2+b^2}$
Damped sine	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
Damped cosine	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
Diverging sine	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
Diverging cosine	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$

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8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0-) - f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right] = \frac{F(s)}{s}$	Integration theorem
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Table 2.2