Mechanical Engineering Department
University of New Mexico

Ph.D. Qualifying Examination

Controls Section

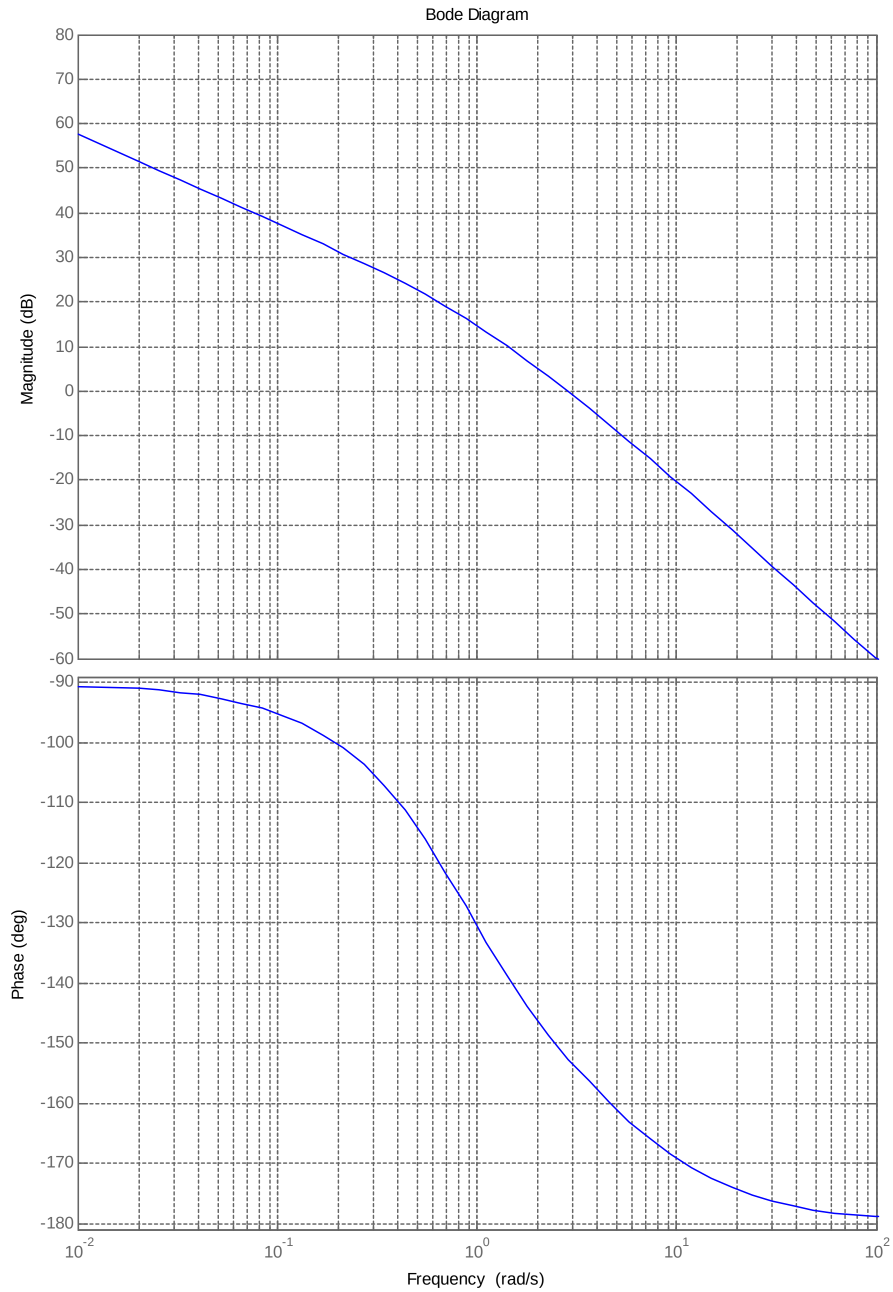
Spring
2021

INSTRUCTIONS:

- Closed Book
- There are 4 problems worth a total of 100 points
- 2 Hours
- Calculator allowed
- You ***MUST*** show work to get credit

Problem 1 (25 pts)

The Bode plot of a plant $G(s)$ is given below.



Consider the closed loop system of placing $G(s)$ in a unity feedback and answer the following.

a) The range of k, placed on the feedforward line as a proportional controller, using which the closed loop system is stable is given by ……………

b) The maximum delay introduced in the control loop before destabilizing the system is …………..

c) The damping ratio of the closed loop systems is roughly equal to ……………

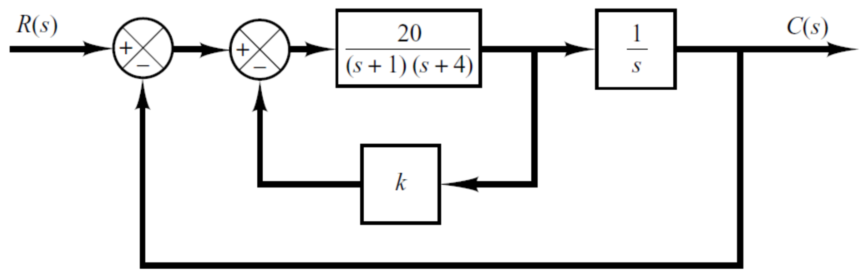
d) The bandwidth of the closed loop system is roughly equal to …………..

e) The steady state error of the closed loop system to a unit step is ………………

f) The steady state error of the closed loop system to a ramp is ………………

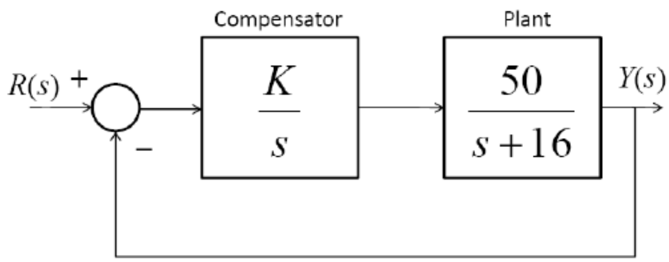
Problem 2 (25 pts)

Sketch (roughly) the location of the poles of the overall system given below, as k changes from zero to infinity.



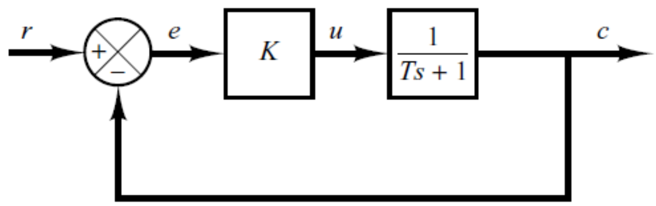
Problem 3 (25 points)

For the unity feedback system shown below,
(a) specify the gain K of the compensator so that the overall closed-loop response to a unit-step input has a damping ratio of 0.517,
(b) for the value of K in (a) compute the peak-time, T_p, and damped frequency, ω_d.



Problem 4. (25 points)

Consider the plant in the figure. Consider two reference signals: $r_1(t) = 1(t)$, the unit-step function, and $r_2(t) = t*1(t)$, the ramp function. Can you quantify the steady state errors in both cases?



Name	Time function, $f(t)$	Laplace transform, $F(s)$
Unit impulse	$\delta(t)$	1
Unit step	$u(t)$	$\frac{1}{s}$
Unit ramp	t	$\frac{1}{s^2}$
nth-order ramp	t^n	$\frac{n!}{s^{n+1}}$
Exponential	e^{-at}	$\frac{1}{s+a}$
nth-order exponential	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
Sine	$\sin bt$	$\frac{b}{s^2+b^2}$
Cosine	$\cos bt$	$\frac{s}{s^2+b^2}$
Damped sine	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
Damped cosine	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
Diverging sine	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
Diverging cosine	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$

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TABLE 2.2 Laplace transform theorems

Item no.	Theorem		Name
1.	$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)]$	$= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)]$	$= F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s) - sf(0-) - f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right]$	$= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty)$	$= \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+)$	$= \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1]For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts, and no more than one can be at the origin.

[2]For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (that is, no impulses or their derivatives at $t = 0$).

Table 2.2

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