Mechanical Engineering Department
University of New Mexico

Ph.D. Qualifying Examination

Controls Section

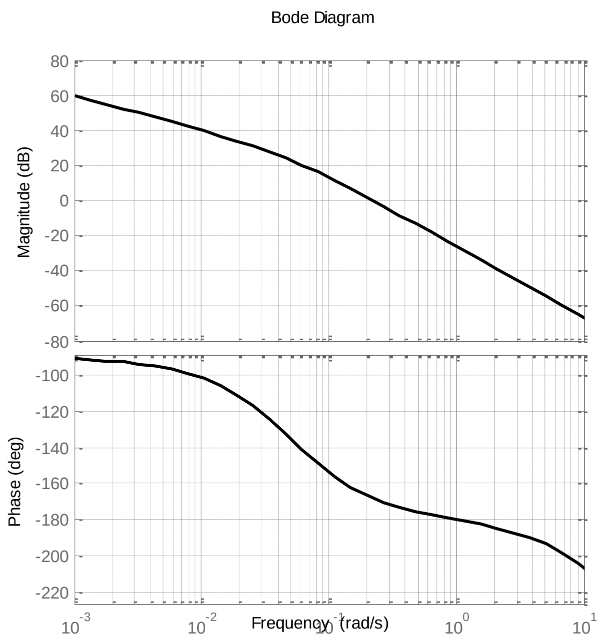
July, 2022

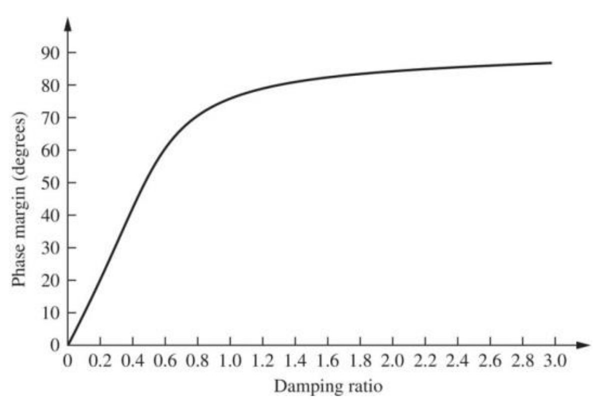
INSTRUCTIONS:

- Closed Book
- There are 4 problems worth a total of 100 points
- 2 Hours
- Calculator allowed
- You ***MUST*** show work to get credit

Problem 1 (25 points)

Using the open loop plant whose Bode plot is given below, design a **Lag Compensator** such that the performance of the closed loop system meets both of the following metrics: a) the steady state error to a ramp is decreased to 1/10 of the uncompensated steady state error, b) The damping ratio of the system is around 0.4.





Problem 2 (25 points)

Match the transfer function on the left with its unit step response on the right.

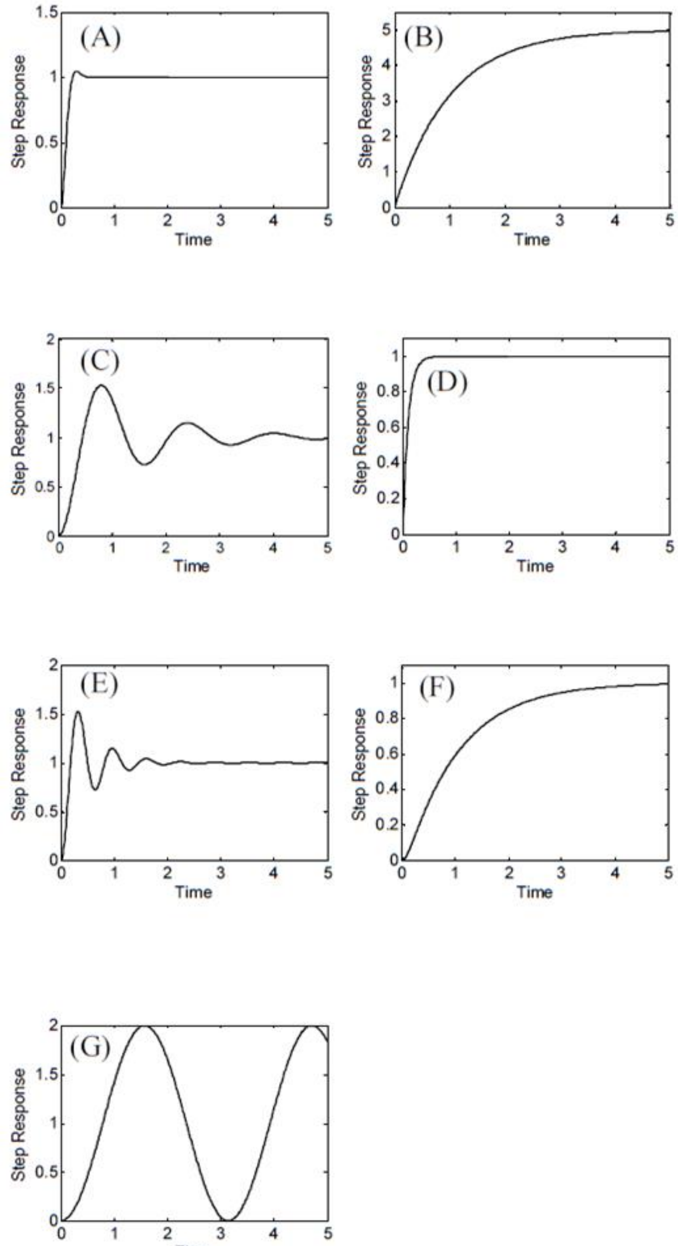
____ (i) $\dfrac{5}{s+1}$

____ (ii) $\dfrac{1}{s^2+4}$

____ (iii) $\dfrac{14^2}{s^2+2\cdot 0.7\cdot 14s+14^2}$

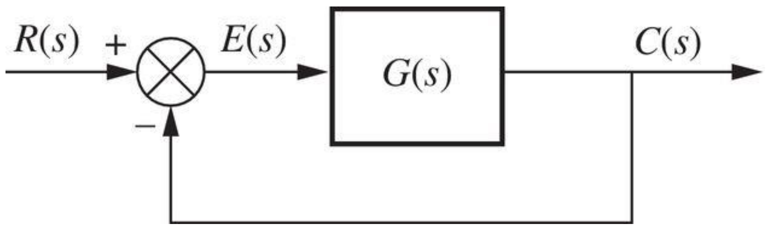
____ (iv) $\dfrac{16}{s^2+1.6s+16}$

____ (v) $\dfrac{10}{(s+10)}$



Problem 3 (25 points)

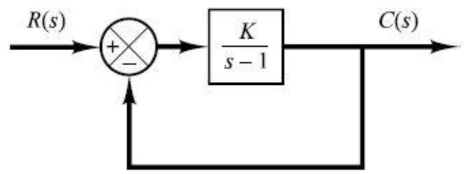
For the given unity feedback system find the range of K that keeps the system stable.



$$G(s) = \frac{K(s+4)}{s(s+1.2)(s+2)}$$

Problem 4 (25 points)

Consider the closed-loop system in the figure. Determine the critical value of K for stability using Nyquist stability criterion.



Name	Time function, $f(t)$	Laplace transform, $F(s)$
Unit impulse	$\delta(t)$	1
Unit step	$u(t)$	$\frac{1}{s}$
Unit ramp	t	$\frac{1}{s^2}$
nth-order ramp	t^n	$\frac{n!}{s^{n+1}}$
Exponential	e^{-at}	$\frac{1}{s+a}$
nth-order exponential	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
Sine	$\sin bt$	$\frac{b}{s^2+b^2}$
Cosine	$\cos bt$	$\frac{s}{s^2+b^2}$
Damped sine	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
Damped cosine	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
Diverging sine	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
Diverging cosine	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$

12

TABLE 2.2 Laplace transform theorems

Item no.	Theorem	Name
1.	$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)] = kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)] = F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)] = F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)] = e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s)-f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s)-sf(0-)-f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s)-\sum_{k=1}^{n}s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right] = \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) = \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+) = \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1]For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts, and no more than one can be at the origin.

[2]For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (that is, no impulses or their derivatives at $t = 0$).

Table 2.2

13