Mechanical Engineering Department

University of New Mexico

Ph.D. Qualifying Examination

Controls Section

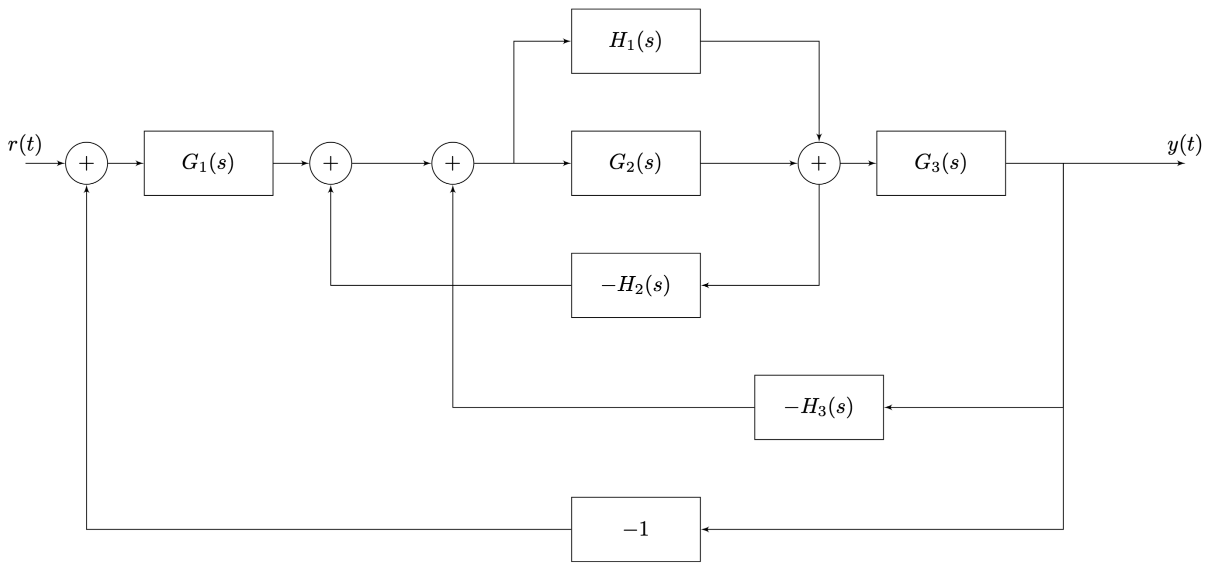
2025

INSTRUCTIONS:

- Time allowed: 2 hours
- Closed book
- Calculator allowed
- Laptops, tablets, cell phones, smart watches, and similar electronic devices are not allowed.
- There are 4 problems worth a total of 100 points

Problem 1 (25 points)

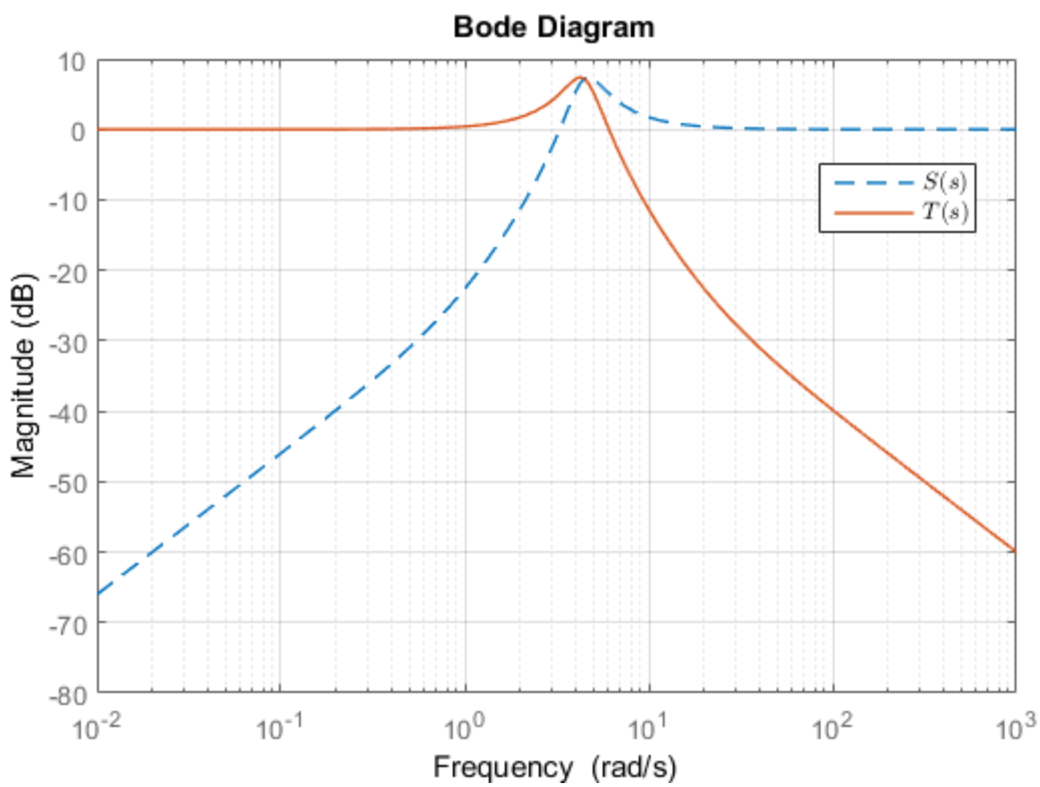
Simplify the block diagram shown below and find the closed loop transfer function from $r(t) \rightarrow y(t)$.



Problem 2 (25 points)

Given a Bode magnitude plot for a unity feedback system showing the sensitivity function $S(s)$ and the complementary sensitivity function $T(s)$, answer the following questions.

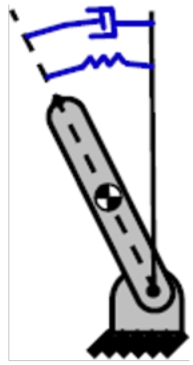
1. Using the plot, give an estimate of the minimum frequency ω_0 such that all noise with frequencies greater than ω_0 appears at the output attenuated to 0.01 times its original magnitude or less.
2. Give an estimate of the maximum frequency ω_1 such that all output disturbance with frequencies less than ω_1 appears at the output attenuated to 0.01 times its original magnitude or less.
3. Suppose that $S(s)$ keeps decreasing in the same fashion as ω approaches 0 rad/sec. Is the loop transfer function Type 0, Type 1 or Type 2?



Problem 3 (25 points)

Given the differential equations below that describe an inverted pendulum with a nonlinear spring, viscous friction, and a motor that applies torque,

$$I\ddot{\theta} + b\dot{\theta} + mgLsin(\theta) + k(1 + a^2\theta^2)\theta = \tau$$



Please do the following:

A) Find a linearized state-space representation of this system with states θ and θ for the equilibrium point when these both equal zero and no torque is being applied.

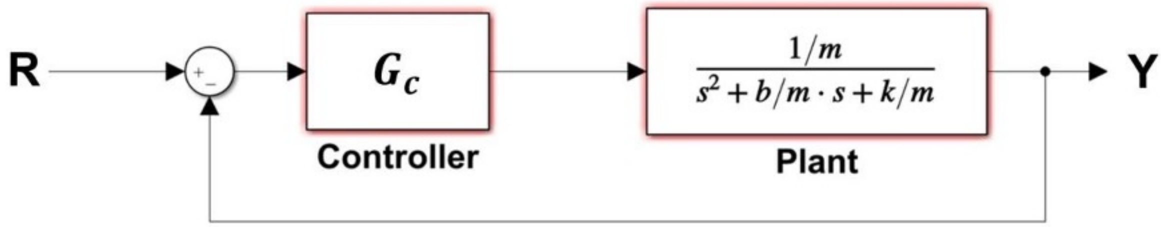
B) Perform a stability analysis on the linearized system and also show if the system is controllable.

C) Explain conceptually what the stability of the linearized system tells us about the original nonlinear system.

D) Using full-state feedback for the linearized system, assuming it is an underdamped system, find a set of gains (K), that will improve the rise time of the original system to be twice as fast.

Problem 4 (25 points)

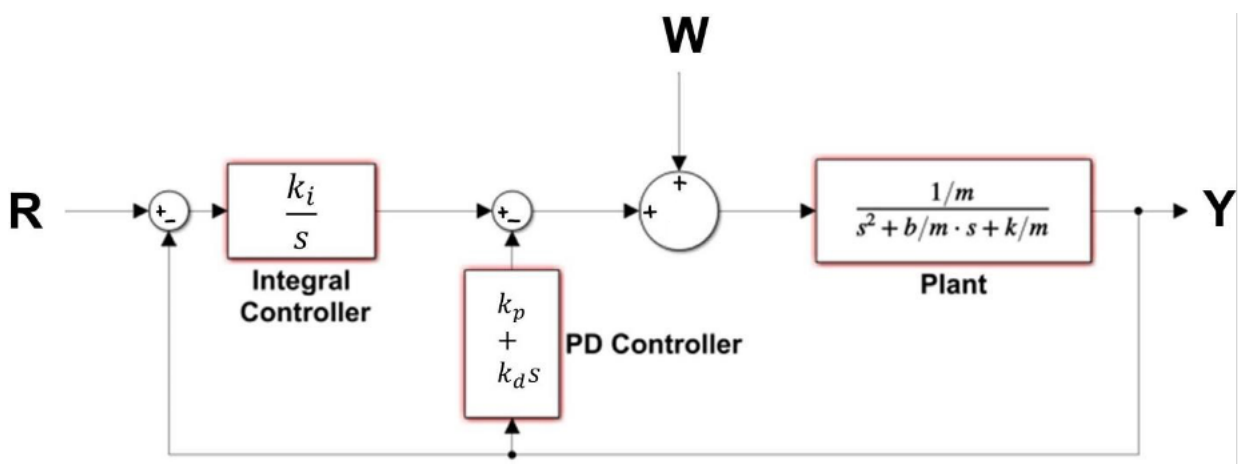
Given the following block diagram and transfer function for the plant:



A) If we let the controller G_c be a simple PD controller, what is our transfer function between the reference input (R) and the plant output (Y)?

B) Neglecting the effect of any zeros in the transfer function from part a), pick gains (k_p and k_d) that will give a rise time of 0.09 sec and a settling time of 0.46 sec if m = 1kg, b = 10 Ns/m, and k = 100 N/m.

C) If instead we chose the following block diagram for our controller architecture (where W is a disturbance input), what will our steady state value of y be for a unit-step input in the reference r? What about for a unit step in w?



Name	Time function, $f(t)$	Laplace transform, $F(s)$
Unit impulse	$\delta(t)$	1
Unit step	$u(t)$	$\frac{1}{s}$
Unit ramp	t	$\frac{1}{s^2}$
nth-order ramp	t^n	$\frac{n!}{s^{n+1}}$
Exponential	e^{-at}	$\frac{1}{s+a}$
nth-order exponential	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
Sine	$\sin bt$	$\frac{b}{s^2+b^2}$
Cosine	$\cos bt$	$\frac{s}{s^2+b^2}$
Damped sine	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
Damped cosine	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
Diverging sine	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
Diverging cosine	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$

TABLE 2.2 Laplace transform theorems

Item no.	Theorem	Name
1.	$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)] = kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)] = F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)] = F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)] = e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s)-f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s)-sf(0-)-f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s)-\sum_{k=1}^{n}s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right] = \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) = \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+) = \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1]For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts, and no more than one can be at the origin.

[2]For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (that is, no impulses or their derivatives at $t = 0$).

Table 2.2