Ph.D. Qualifying Examination Controls
July 10 2024

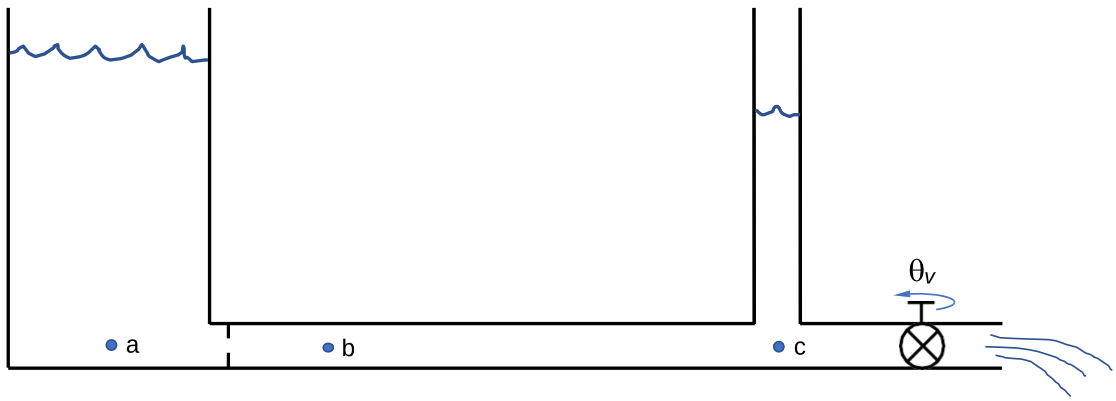
Logistics Notes

- Time allowed: 2.5 hours
- Closed book and closed notes; one sheet (8:5 × 11 in, 2-sided) of formulas is allowed
- Four problems
- Calculators are allowed
- Laptops, cell phones, and similar electronic devices with Internet access are not allowed

Show your work, including intermediate steps. State your assumptions clearly. Use as many sheets of paper as necessary to present each solution.

Problem 1: Modeling of fluid system

You are to model the dynamics of the fluid system shown below. Distinct pressures for this system are denoted by the points *a*, *b*, and *c* in the diagram. You are to model the fluid capacitance effects of the two tanks, the resistance of the orifice in the line, the fluid inertia in the line, and the resistance of the valve. For your model, you should assume that the fluid line is rigid and the fluid is not compressible. Derive the equations of motion. The resulting equations will have two pressure states and one flow state.

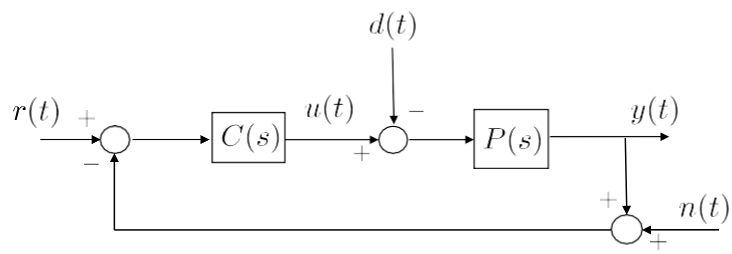


If you were interested in simulating the height of the fluid in the tanks, what would be your approach to do that?

Problem 2 - frequency-response-based control design:

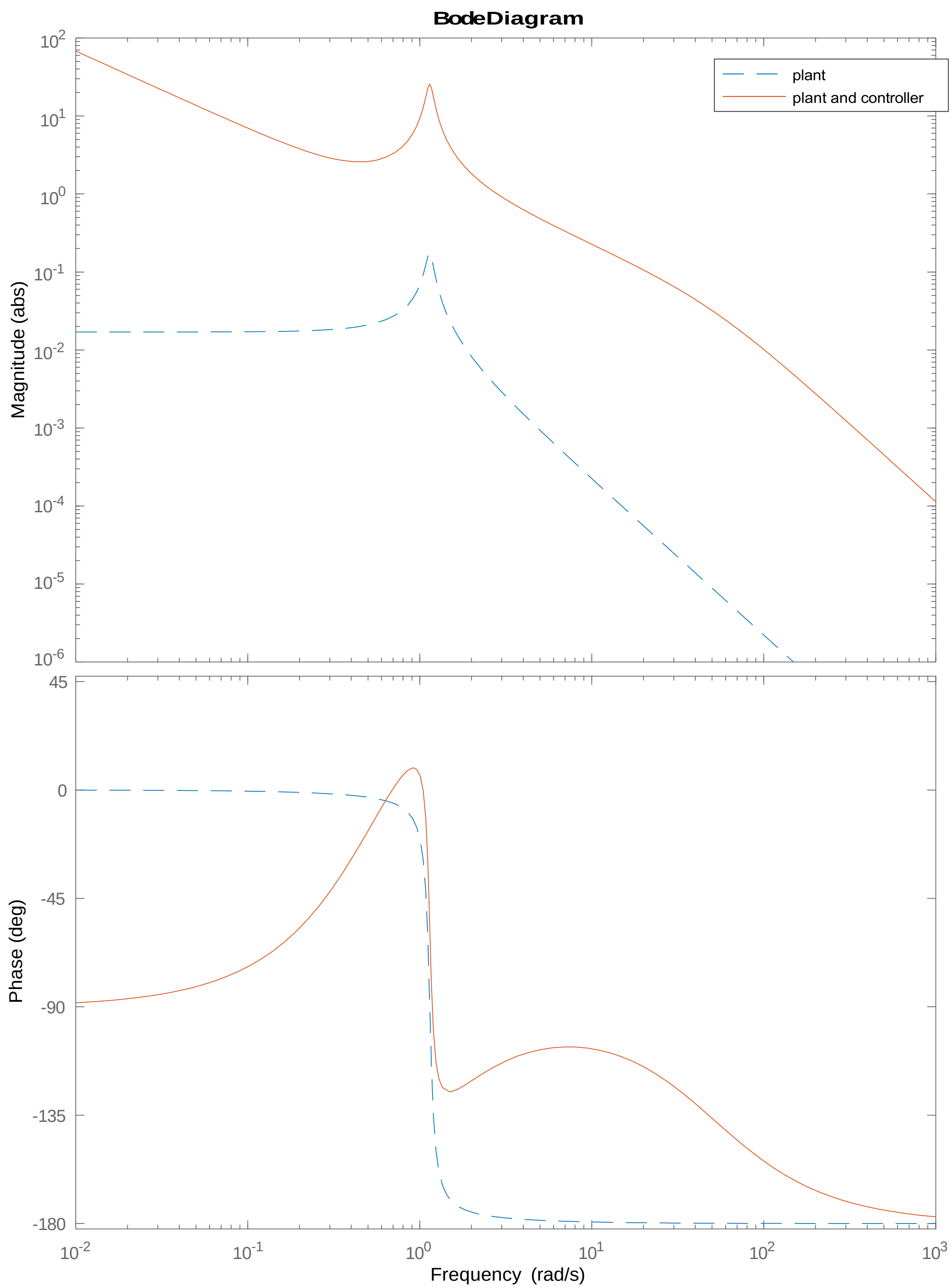
The rolling motion of an underwater vehicle is controlled by a PID controller of the form

The control system has the following structure where $P(s)$ is the transfer function of the plant.



The *open-loop frequency response* for the plant $P(s)$ and the combined controller and plant $C(s)P(s)$ are shown on the following page. From these plots and the information above, determining the following:

(1) Estimate the anticipated rise time for the closed-loop system in response to a step input in $r(t)$.
(2) Estimate of the anticipated percent overshoot for the closed-loop system in response to a step input in $r(t)$.
(3) Estimate the steady-state error of the system in response to a ramp input in $r(t)$.
(4) For a disturbance input given by $d(t) = 3\sin(0.2t)$, estimate the magnitude of the output $y(t)$.
(5) If noise is introduced by a sensor measurement that has a magnitude of 2 degrees, what is the lowest noise frequency that can be tolerated if the effect of the noise on the output must be kept below 0.01 degrees?

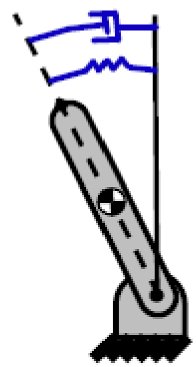


Bode Diagram
plant
plant and controller
Magnitude (abs)
Phase (deg)
Frequency (rad/s)
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
45
0
-45
-90
-135
-180
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

Problem 3 – linearized dynamics

Given the differential equations below that describe an inverted pendulum with a nonlinear spring, viscous friction, and a motor that applies torque,

$$I\ddot{\theta} + b\dot{\theta} + mgLsin(\theta) + k(1 + a^2\theta^2)\theta = \tau$$

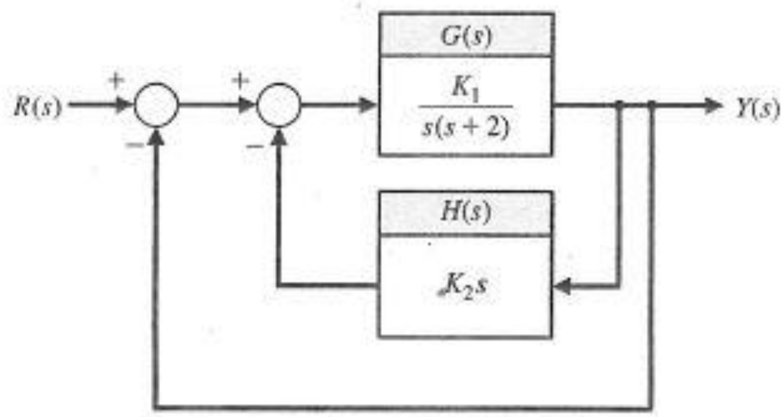


please do the following:

a) Find a linearized state-space representation of this system with states θ and $\dot{\theta}$ for the equilibrium point when these both equal zero and no torque is being applied.
b) Perform a stability analysis on the linearized system and also show if the system is controllable.
c) Explain conceptually what the stability of the linearized system tells us about the original nonlinear system.
d) Using full-state feedback for the linearized system, assuming it is an underdamped system, find a set of gains (K), that will improve the rise time of the original system to be twice as fast.

Problem 4: Root Locus

Consider the feedback system in the figure, representing an accurate control system for positioning a welding head:



Use the root locus to select the amplifier gain K_1 and the derivative feedback gain K_2 so that the following specifications are satisfied:

1) Steady state error for a unit ramp $e_{ss} \leq 0.35$
2) Damping ratio of dominant root $\zeta \geq 0.707$
3) Settling time to within 2% of the final value is equal to $T_s \leq 3$s