# University of New Mexico Mechanical engineering Spring 2016 Ph.D. qualifying examination

# Heat Transfer

#### Notes

- Time allowed: 150 minutes.
- Closed book / Closed Notes (one  $8.5 \times 11.00$  in. sheet of formulas is allowed).
- Calculators are allowed.
- Laptops, cell phones, and similar electronic devices are not allowed.

## Part 1: General knowledge questions (25 points)

Circle the appropriate answer. No penalty for wrong answer.

- 1. Newton's law of cooling states that:
  - (a) The net force applied to an object is proportional to the rate of change of its momentum.
  - (b) Heat flux in some direction is proportional to the gradient in temperature in the same direction.
  - (c) The rate of cooling of an object is proportional to the difference in temperature between the object and the ambient.
- 2. If a basketball ball is heat and players are atoms, then convective heat transfer is analogous to:
  - (a) A player throwing the ball into the hoop.
  - (b) Players passing the ball to each other.
  - (c) A player dribbling the ball and moving towards the hoop.
- 3. Natural and forced convection differ in that
  - (a) natural convection is found in natural processes, forced convection only in artificial processes.
  - (b) Natural convection is always stronger than forced convection.
  - (c) Natural convection is driven by buoyancy, forced convection is driven by a flow imposed by forces external to the convective process.
- 4. Arrays of vertical fins are used to remove heat from devices such as electronic equipment. For vertical fins in natural convection, there exists an optimum fin spacing, which represents the best compromise between:
  - (a) The weight and the size of the array.
  - (b) The reduction in convection coefficient with more closely spaced fins and the increase in total surface area.
  - (c) The increase in convection coefficient and the increase in weight with more closely spaced fins.
- 5. A good example of the application of natural convection is:
  - (a) To cool automobile engines, thereby removing the necessity for fans.
  - (b) To cool certain types of nuclear reactor cores, so we don't have to rely on pumps working.
  - (c) To remove heat from a house in hot summer months.

- 6. Typical engineered phase-change cooling systems operate in:
  - (a) the nucleate boiling regime, ensuring high cooling capacity, low excess temperatures, and a margin of safety.
  - (b) at the critical point, ensuring maximum possible performance with small excess temperature.
  - (c) In the film boiling regime, ensuring the combination of high excess temperature and high heat removal rates.
- 7. The fundamental difference between convection processes in tube flow and in external flow is that:
  - (a) In external flow, heat is always removed at a greater rate.
  - (b) In external flow, there is an infinite (for all practical purposes) supply of fluid at constant temperature, while in tube flow the bulk temperature of the fluid is affected by the convective process.
  - (c) in tube flow, the Nusselt number is constant, in external flow it is not.
- 8. A selective surface is one in which:
  - (a) emissivity and absorptivity depend on wavelength but are equal to each other.
  - (b) emissivity and absorptivity at a given wavelength can be different.
  - (c) the surface emits radiation only in certain directions.
- 9. The view factor  $F_{ij}$  between two objects *i* and *j* is:
  - (a) the fraction of radiant energy emitted by i which impinges on j.
  - (b) the fraction of radiant energy emitted by j which impinges on i.
  - (c) the solid angle subtended by j as seen from i.
- 10. A large cavity with a small opening is approximately equivalent to a black surface because
  - (a) it looks dark.
  - (b) a photon entering the cavity is very unlikely to come back out.
  - (c) no radiation ever emerges from inside the cavity to the outside.

## Part 2: Problems (25 points per question)

Attempt all problems in this section, clearly stating any assumptions and simplifications used in your solution.

#### Problem 1



A power transmission cable can be approximated as a horizontal cylinder. To prevent excessive sagging, the cable cannot become hotter than 70°C. Each cable in the power line in question has a resistance of 0.127  $\Omega/\text{km}$ , and a diameter of 12 mm. For the maximum allowable temperature, calculate:

- 1. the convection coefficient for a calm summer day, with an air temperature of 35°C;
- 2. the heat loss from the cable to the surrounding air per unit length of cable;
- 3. The current in the line (recall: resistive power loss =  $I^2 R$ ).

#### Problem 2



A heat exchanger is composed of a thin-walled copper tube, of diameter 14 mm, immersed in an unpressurized container in which a water/ice mixture is stirred vigorously. Air at atmospheric pressure enters the copper tube at 40°C, with a flow rate of 0.01 m<sup>3</sup>/s. It is required that the air exits the tube at a temperature no higher than 5°C. Calculate:

- 1. the convection coefficient for heat flow from the air to the tube walls;
- 2. the overall heat transfer coefficient U for the heat transfer from the air to the water/ice mixture, if the convection coefficient outside the tube is 400 W/m<sup>2</sup>·K;
- 3. the minimum length of the immersed part of the copper tube;
- 4. the heat transfer rate from the air to the water/ice mixture.

#### Problem 3



A radiating surface, to be used to reject heat to the night sky, is coated with a material which has an emissivity of 0.9. The surface is dimpled to improve its capacity to release heat. The dimples are arranged in a square packing, with a center-to-center distance of 10 mm, a radius of 4 mm, and a depth of 5 mm. During a typical night, the surface temperature is  $20^{\circ}$  C, while the sky temperature is  $-50^{\circ}$  C. Calculate:

- 1. The ratio of dimpled surface to total surface, for a unit area of material;
- 2. The view factor from the dimple to the sky;
- 3. The amount of heat released to the sky by the dimple area;
- 4. The amount of heat released to the sky by the flat area;
- 5. The improvement in the heat radiation capacity for the dimpled surface with respect to a flat surface of the same overall dimensions.

#### Tables & other useful information

Temp. <i>T</i> , °C	Density $ ho$ , kg/m <sup>3</sup>	Specific Heat c <sub>p</sub> , J∫kg ⋅ K	Thermal Conductivity <i>k</i> , W/m · K	Thermal Diffusivity $\alpha$ , m <sup>2</sup> /s <sup>2</sup>	Dynamic Viscosity µ, kg/m · s	Kinematic Viscosity $\nu$ , m <sup>2</sup> /s	Prandtl Number Pr
-150	2.866	983	0.01171	$4.158 \times 10^{-6}$	$8.636 \times 10^{-6}$	2012 × 10-6	0.70
-100	2.038	966	0.01582	$4.130 \times 10^{-6}$	$1.180 \times 10^{-5}$	$5.015 \times 10^{\circ}$	0.7246
-50	1.582	999	0.01979	$1.252 \times 10^{-5}$	$1.109 \times 10^{-5}$ $1.474 \times 10^{-5}$	$0.310 \times 10^{-6}$	0.7263
-40	1.514	1002	0.02057	$1.252 \times 10^{-5}$ 1.356 × 10 <sup>-5</sup>	$1.474 \times 10^{-5}$	$9.319 \times 10^{-5}$	0.7440
-30	1.451	1004	0.02134	$1.650 \times 10^{-5}$ 1.465 × 10 <sup>-5</sup>	$1.527 \times 10^{-5}$	$1.000 \times 10^{-5}$	0.7436
-20	1.394	1005	0.02211	$1.700 \times 10^{-5}$ 1.578 × 10 <sup>-5</sup>	$1.630 \times 10^{-5}$	$1.067 \times 10^{-5}$	0.7425
-10	1.341	1006	0.02288	$1.696 \times 10^{-5}$	$1.680 \times 10^{-5}$	$1.109 \times 10^{-5}$ $1.252 \times 10^{-5}$	0.7408
0	1.292	1006	0.02364	$1.818 \times 10^{-5}$	$1.000 \times 10^{-5}$ 1.729 × 10 <sup>-5</sup>	$1.232 \times 10^{-5}$	0.738/
5	1.269	1006	0.02401	$1.880 \times 10^{-5}$	$1.723 \times 10^{-5}$ 1.754 × 10 <sup>-5</sup>	$1.382 \times 10^{-5}$	0.7362
10	1.246	1006	0.02439	$1.944 \times 10^{-5}$	$1.734 \times 10^{-5}$	$1.302 \times 10^{-5}$	0.7350
15	1.225	1007	0.02476	$2.009 \times 10^{-5}$	$1.802 \times 10^{-5}$	$1.420 \times 10^{-5}$	0.7336
20	1.204	1007	0.02514	$2.074 \times 10^{-5}$	$1.802 \times 10^{-5}$	$1.470 \times 10^{-5}$ 1.516 × 10 <sup>-5</sup>	0.7323
25	1.184	1007	0.02551	$2.141 \times 10^{-5}$	$1.849 \times 10^{-5}$	$1.510 \times 10^{-5}$ $1.562 \times 10^{-5}$	0.7309
30	1.164	1007	0.02588	$2.208 \times 10^{-5}$	$1.872 \times 10^{-5}$	$1.502 \times 10^{-5}$	0.7296
35	1.145	1007	0.02625	$2.277 \times 10^{-5}$	$1.895 \times 10^{-5}$	$1.655 \times 10^{-5}$	0.7282
40	1.127	1007	0.02662	$2.346 \times 10^{-5}$	$1.030 \times 10^{-5}$ 1.918 × 10 <sup>-5</sup>	$1.000 \times 10^{-5}$	0.7268
45	1.109	1007	0.02699	$2.416 \times 10^{-5}$	$1.941 \times 10^{-5}$	$1.762 \times 10^{-5}$	0.7255
50	1.092	1007	0.02735	$2.487 \times 10^{-5}$	$1.963 \times 10^{-5}$	$1.798 \times 10^{-5}$	0.7241
60	1.059	1007	0.02808	$2.632 \times 10^{-5}$	$2.008 \times 10^{-5}$	$1.896 \times 10^{-5}$	0.7228
70	1.028	1007	0.02881	$2.780 \times 10^{-5}$	$2.052 \times 10^{-5}$	$1.090 \times 10^{-5}$ 1.995 × 10 <sup>-5</sup>	0.7202
80	0.9994	1008	0.02953	$2.931 \times 10^{-5}$	$2.096 \times 10^{-5}$	$2.097 \times 10^{-5}$	0.7177
90	0.9718	1008	0.03024	$3.086 \times 10^{-5}$	$2.139 \times 10^{-5}$	$2.007 \times 10^{-5}$	0.7134
100	0.9458	1009	0.03095	$3.243 \times 10^{-5}$	$2.181 \times 10^{-5}$	$2.306 \times 10^{-5}$	0.7132
120	0.8977	1011	0.03235	$3.565 \times 10^{-5}$	$2.264 \times 10^{-5}$	$2.500 \times 10^{-5}$	0.7073
140	0.8542	1013	0.03374	$3.898 \times 10^{-5}$	$2.345 \times 10^{-5}$	$2.022 \times 10^{-5}$ 2.745 × 10 <sup>-5</sup>	0.7073
160	0.8148	1016	0.03511	$4.241 \times 10^{-5}$	$2.420 \times 10^{-5}$	$2.975 \times 10^{-5}$	0.7014
180	0.7788	1019	0.03646	$4.593  imes 10^{-5}$	$2.504 \times 10^{-5}$	$3.212 \times 10^{-5}$	0.6992
200	0.7459	1023	0.03779	$4.954  imes 10^{-5}$	$2.577 \times 10^{-5}$	$3.455 \times 10^{-5}$	0.6974
250	0.6746	1033	0.04104	$5.890  imes 10^{-5}$	$2.760 \times 10^{-5}$	$4.091 \times 10^{-5}$	0.6946
300	0.6158	1044	0.04418	$6.871  imes 10^{-5}$	$2.934 \times 10^{-5}$	$4.765 \times 10^{-5}$	0.6935
350	0.5664	1056	0.04721	$7.892  imes 10^{-5}$	$3.101 \times 10^{-5}$	$5.475 \times 10^{-5}$	0.6937
400	0.5243	1069	0.05015	$8.951  imes 10^{-5}$	$3.261 \times 10^{-5}$	$6.219 \times 10^{-5}$	0.6948
450	0.4880	1081	0.05298	$1.004 \times 10^{-4}$	$3.415 \times 10^{-5}$	$6.997 \times 10^{-5}$	0.6965
500	0.4565	1093	0.05572	$1.117  imes 10^{-4}$	$3.563  imes 10^{-5}$	$7.806 \times 10^{-5}$	0.6986
600	0.4042	1115	0.06093	$1.352  imes 10^{-4}$	$3.846  imes 10^{-5}$	$9.515 \times 10^{-5}$	0.7037
/00	0.3627	1135	0.06581	$1.598 imes10^{-4}$	$4.111  imes 10^{-5}$	$1.133 \times 10^{-4}$	0.7092
800	0.3289	1153	0.07037	$1.855  imes 10^{-4}$	$4.362  imes 10^{-5}$	$1.326 \times 10^{-4}$	0.7149
900	0.3008	1169	0.07465	$2.122 \times 10^{-4}$	$4.600  imes 10^{-5}$	$1.529 \times 10^{-4}$	0.7206
1000	0.2772	1184	0.07868	$2.398  imes 10^{-4}$	$4.826  imes 10^{-5}$	$1.741 \times 10^{-4}$	0.7260
1500	0.1990	1234	0.09599	$3.908  imes 10^{-4}$	$5.817 \times 10^{-5}$	$2.922 \times 10^{-4}$	0.7478
2000	0.1553	1264	0.11113	$5.664  imes 10^{-4}$	$6.630  imes 10^{-5}$	$4.270  imes 10^{-4}$	0.7539

Properties of air at 1 atm pressure

*Note*: For ideal gases, the properties  $c_{\rho}$ , k,  $\mu$ , and Pr are independent of pressure. The properties  $\rho$ ,  $\nu$ , and  $\alpha$  at a pressure P (in atm) other than 1 atm are determined by multiplying the values of  $\rho$  at the given temperature by P and by dividing  $\nu$  and  $\alpha$  by P.

Source: Data generated from the EES software developed by S. A. Klein and F. L. Alvarado. Original sources: Keenan, Chao, Keyes, Gas Tables, Wiley, 198; and Thermophysical Properties of Matter. Vol. 3: Thermal Conductivity, Y. S. Touloukian, P. E. Liley, S. C. Saxena, Vol. 11: Viscosity, Y. S. Touloukian, S. C. Saxena, and P. Hestermans, IFI/Plenun, NY, 1970, ISBN 0-306067020-8.

#### Properties of saturated water

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Temp.	Saturation Pressure	[ F	Density o, kg/m <sup>3</sup>	Enthalpy of Vaporizatior	Speci He n <u>c<sub>p</sub>, J/</u>	fic at kg · K	The Condu k, W	ermal ictivity /m · K	Dynami , k	c Viscosity g/m · s	Pra Nur F	indtl mber Pr	Volume Expansion Coefficient β, 1/K
<i>T</i> , °C	P <sub>sat</sub> , kPa	Liquio	d Vapor	h <sub>fg</sub> , kJ/kg	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid
0.01	0.6113	999.8	0.0048	2501	4217	1854	0.561	0.0171	$1.792 \times 10^{-3}$	$0.922 \times 10^{-5}$	13.5	1.00	-0.068 × 10-
5	0.8721	999.9	0.0068	2490	4205	1857	0.571	0.0173	$1.519 imes10^{-3}$	$0.934  imes 10^{-5}$	11.2	1.00	0.015 × 10-3
10	1.2276	999.7	0.0094	2478	4194	1862	0.580	0.0176	$1.307 \times 10^{-3}$	$0.946  imes 10^{-5}$	9.45	1.00	0.733 × 10-3
15	1.7051	999.1	0.0128	2466	4185	1863	0.589	0.0179	$1.138  imes 10^{-3}$	$0.959  imes 10^{-5}$	8.09	1.00	0.138 × 10-3
20	2.339	998.0	0.0173	2454	4182	1867	0.598	0.0182	$1.002 \times 10^{-3}$	$0.973  imes 10^{-5}$	7.01	1.00	$0.195 \times 10^{-3}$
25	3.169	997.0	0.0231	2442	4180	1870	0.607	0.0186	$0.891 \times 10^{-3}$	$0.987  imes 10^{-5}$	6.14	1.00	0.247 × 10-3
30	4.246	996.0	0.0304	2431	4178	1875	0.615	0.0189	$0.798  imes 10^{-3}$	$1.001  imes 10^{-5}$	5.42	1.00	0.294 × 10-3
35	5.628	994.0	0.0397	2419	4178	1880	0.623	0.0192	$0.720  imes 10^{-3}$	$1.016  imes 10^{-5}$	4.83	1.00	0.337 × 10-3
40	7.384	992.1	0.0512	2407	4179	1885	0.631	0.0196	$0.653 \times 10^{-3}$	$1.031  imes 10^{-5}$	4.32	1.00	0.377 × 10-3
45	9.593	990.1	0.0655	2395	4180	1892	0.637	0.0200	$0.596 \times 10^{-3}$	$1.046  imes 10^{-5}$	3.91	1.00	$0.415 \times 10^{-3}$
50	12.35	988.1	0.0831	2383	4181	1900	0.644	0.0204	$0.547 \times 10^{-3}$	$1.062 \times 10^{-5}$	3.55	1.00	$0.451 \times 10^{-3}$
55	15.76	985.2	0.1045	23/1	4183	1908	0.649	0.0208	$0.504 \times 10^{-3}$	$1.077 \times 10^{-5}$	3.25	1.00	$0.484 \times 10^{-3}$
65	25.94	903.3	0.1304	2359	4185	1916	0.654	0.0212	$0.467 \times 10^{-3}$	$1.093 \times 10^{-5}$	2.99	1.00	$0.517 \times 10^{-3}$
70	25.05	980.4	0.1014	2346	4187	1926	0.659	0.0216	$0.433 \times 10^{-3}$	$1.110 \times 10^{-5}$	2.75	1.00	$0.548  imes 10^{-3}$
75	31.19	977.5	0.1983	2334	4190	1936	0.663	0.0221	$0.404 \times 10^{-3}$	$1.126 \times 10^{-5}$	2.55	1.00	$0.578  imes 10^{-3}$
80	17 20	974.7	0.2421	2321	4193	1948	0.667	0.0225	$0.378 \times 10^{-3}$	$1.142 \times 10^{-5}$	2.38	1.00	$0.607  imes 10^{-3}$
85	57.83	9/1.0	0.2935	2309	4197	1962	0.670	0.0230	$0.355 \times 10^{-3}$	$1.159 \times 10^{-5}$	2.22	1.00	$0.653  imes 10^{-3}$
90	70.14	900.1	0.3030	2290	4201	19//	0.675	0.0235	$0.333 \times 10^{-3}$	$1.1/6 \times 10^{-5}$	2.08	1.00	$0.670 \times 10^{-3}$
95	84.55	905.5	0.4235	2203	4200	1993	0.675	0.0240	$0.315 \times 10^{-3}$	$1.193 \times 10^{-5}$	1.96	1.00	$0.702 \times 10^{-3}$
100	101 33	957.9	0.5045	2257	4212	2010	0.670	0.0246	$0.297 \times 10^{-3}$	$1.210 \times 10^{-5}$	1.85	1.00	$0.716 \times 10^{-3}$
110	143 27	950.6	0.3978	2230	4217	2029	0.679	0.0251	$0.282 \times 10^{-3}$	$1.227 \times 10^{-5}$	1.75	1.00	$0.750 \times 10^{-3}$
120	198.53	943.4	1 1 2 1	2203	4225	2120	0.002	0.0262	$0.255 \times 10^{-3}$	$1.261 \times 10^{-5}$	1.58	1.00	$0.798 \times 10^{-3}$
130	270.1	934.6	1 496	2174	4244	2120	0.005	0.0275	$0.232 \times 10^{-3}$	$1.296 \times 10^{-5}$	1.44	1.00	$0.858 \times 10^{-3}$
140	361.3	921.7	1.965	2145	4286	221/7	0.084	0.0200	$0.213 \times 10^{-3}$	$1.330 \times 10^{-5}$	1.33	1.01	$0.913 \times 10^{-3}$
150	475.8	916.6	2 546	2114	4200	2244	0.083	0.0301	$0.197 \times 10^{-3}$	$1.365 \times 10^{-5}$	1.24	1.02	$0.970 \times 10^{-3}$
160	617.8	907.4	3.256	2083	4340	2420	0.680	0.0310	$0.183 \times 10^{-3}$	$1.399 \times 10^{-5}$	1.16	1.02	$1.025 \times 10^{-3}$
170	791.7	897.7	4.119	2050	4370	2490	0.677	0.0347	$0.170 \times 10^{-3}$	$1.454 \times 10^{-5}$	1.09	1.05	$1.145 \times 10^{-3}$
180	1,002.1	887.3	5.153	2015	4410	2590	0.673	0.0347	$0.100 \times 10^{-3}$	$1.400 \times 10^{-5}$	1.03	1.05	$1.178 \times 10^{-3}$
190	1,254.4	876.4	6.388	1979	4460	2710	0.669	0.0382	$0.130 \times 10^{-3}$	$1.502 \times 10^{-5}$	0.965	1.07	$1.210 \times 10^{-3}$
200	1,553.8	864.3	7.852	1941	4500	2840	0.663	0.0302	$0.142 \times 10^{-3}$	$1.571 \times 10^{-5}$	0.947	1.09	$1.280 \times 10^{-3}$
220	2,318	840.3	11.60	1859	4610	3110	0.650	0.0401	$0.134 \times 10^{-3}$	$1.571 \times 10^{-5}$	0.910	1.11	$1.350 \times 10^{-3}$
240	3,344	813.7	16.73	1767	4760	3520	0.632	0.0487	$0.122 \times 10^{-3}$	$1.041 \times 10$ $1.712 \times 10^{-5}$	0.805	1.10	$1.320 \times 10^{-3}$
260	4,688	783.7	23.69	1663	4970	4070	0.609	0.0540	$0.102 \times 10^{-3}$	$1.712 \times 10^{-5}$	0.000	1.24	$1.720 \times 10^{-3}$
280	6,412	750.8	33.15	1544	5280	4835	0.581	0.0605	$0.094 \times 10^{-3}$	$1.700 \times 10^{-5}$	0.854	1 / 9	$2.000 \times 10^{-3}$
300	8,581	713.8	46.15	1405	5750	5980	0.548	0.0695	$0.086 \times 10^{-3}$	$1.965 \times 10^{-5}$	0.004	1.49	$2.360 \times 10^{-3}$
320	11,274	667.1	64.57	1239	6540	7900	0.509	0.0836	$0.078 \times 10^{-3}$	$2.084 \times 10^{-5}$	1 00	1.05	2.350 ~ 10
340	14,586	610.5	92.62	1028	8240	11,870	0.469	0.110	$0.070 \times 10^{-3}$	$2.255 \times 10^{-5}$	1.00	2 43	
360	18,651	528.3	144.0	720	14,690	25,800	0.427	0.178	$0.060 \times 10^{-3}$	$2.571 \times 10^{-5}$	2.06	3 73	
374.14	22,090	317.0	317.0	0	_	_	_		$0.043 \times 10^{-3}$	$4.313 \times 10^{-5}$	2.00	5.75	

Note 1: Kinematic viscosity  $\nu$  and thermal diffusivity  $\alpha$  can be calculated from their definitions,  $\nu = \mu/\rho$  and  $\alpha = k/\rho c_{\rho} = \nu/Pr$ . The temperatures 0.01°C, 100°C, and 374.14°C are the triple-, boiling-, and critical-point temperatures of water, respectively. The properties listed above (except the vapor density) can be used at any pressure with negligible error except at temperatures near the critical-point value.

Note 2: The unit kJ/kg · °C for specific heat is equivalent to kJ/kg · K, and the unit W/m · °C for thermal conductivity is equivalent to W/m · K.

Source: Viscosity and thermal conductivity data are from J. V. Sengers and J. T. R. Watson, Journal of Physical and Chemical Reference Data 15 (1986), pp. 1291–1322. Other data are obtained from various sources or calculated.

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Geometry	Characteristic length $L_c$	Range of Ra	Nu	
Vertical plate	L	10 <sup>4</sup> –10 <sup>9</sup> 10 <sup>10</sup> –10 <sup>13</sup> Entire range	$\begin{split} Ν = 0.59Ra_{\textit{L}}^{1/4} \\ Ν = 0.1Ra_{\textit{L}}^{1/3} \\ Ν = \left\{ 0.825 + \frac{0.387Ra_{\textit{L}}^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \right\}^2 \\ & \text{(complex but more accurate)} \end{split}$	(9–19) (9–20) (9–21)
Inclined plate	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos\theta$ for Ra $< 10^9$	
Horizontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate) Hot surface $T_s$	A <sub>s</sub> /p	10 <sup>4</sup> -10 <sup>7</sup> 10 <sup>7</sup> -10 <sup>11</sup>	$Nu = 0.54Ra_{L}^{1/4}$ $Nu = 0.15Ra_{L}^{1/3}$	(9–22) (9–23)
(or upper surface of a cold plate) $T_{s}$ Hot surface		10 <sup>5</sup> -10 <sup>11</sup>	$Nu = 0.27 Ra_L^{1/4}$	(9–24)
Vertical cylinder	L		A vertical cylinder can be treated as a vertical plate when $D \ge \frac{35L}{\text{Gr}_{L}^{1/4}}$	
Horizontal cylinder	D	$Ra_D \leq 10^{12}$	$Nu = \left\{ 0.6 + \frac{0.387 \text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}} \right\}^2$	(9–25)
Sphere	D	$Ra_D \le 10^{11}$ (Pr $\ge 0.7$ )	$Nu = 2 + \frac{0.589 \text{Ra}^{3/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$	(9–26)

Empirical correlations for the average Nusselt number for natural convection over surfaces

1011	alh	Nusse	Friction Factor			
Tube Geometry	or $\theta^{\circ}$	$T_s = \text{Const.}$	$\dot{q}_s = \text{Const.}$	f		
Circle		3.66	4.36	64.00/Re		
Rectangle	<i><u>alb</u></i> 1 2 3 4 6 8 ∞	2.98 3.39 3.96 4.44 5.14 5.60 7.54	3.61 4.12 4.79 5.33 6.05 6.49 8.24	56.92/Re 62.20/Re 68.36/Re 72.92/Re 78.80/Re 82.32/Re 96.00/Re		
	<i><u>a/b</u> 1 2 4 8 16</i>	3.66 3.74 3.79 3.72 3.65	4.36 4.56 4.88 5.09 5.18	64.00/Re 67.28/Re 72.96/Re 76.60/Re 78.16/Re		
Isosceles Triangle	$ \begin{array}{c} \frac{\theta}{10^{\circ}} \\ 30^{\circ} \\ 60^{\circ} \\ 90^{\circ} \\ 120^{\circ} \end{array} $	1.61 2.26 2.47 2.34 2.00	2.45 2.91 3.11 2.98 2.68	50.80/Re 52.28/Re 53.32/Re 52.60/Re 50.96/Re		

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ( $D_h = 4A_c/p$ , Re =  $V_{avg}D_h/\nu$ , and Nu =  $hD_h/k$ )

For fully developed laminar flow in a circular pipe, we have

$$u(r) = 2V_{\text{avg}}\left(1 - \frac{r^2}{R^2}\right) = u_{\text{max}}\left(1 - \frac{r^2}{R^2}\right)$$
$$f = \frac{64\mu}{\rho D V_{\text{avg}}} = \frac{64}{\text{Re}}$$
$$\dot{V} = V_{\text{avg}}A_c = \frac{\Delta P R^2}{8\mu L} \pi R^2 = \frac{\pi R^4 \Delta P}{8\mu L} = \frac{\pi R^4 \Delta P}{128\mu L}$$

Circular tube, laminar ( $\dot{q}_s = \text{constant}$ ): Nu =  $\frac{hD}{k} = 4.36$ Circular tube, laminar ( $T_s = \text{constant}$ ): Nu =  $\frac{hD}{k} = 3.66$ 

For *developing laminar flow* in the entrance region with constant surface temperature, we have

Circular tube: Nu =  $3.66 + \frac{0.065(D/L) \text{ Re Pr}}{1 + 0.04[(D/L) \text{ Re Pr}]^{2/3}}$ Circular tube: Nu =  $1.86 \left(\frac{\text{Re Pr }D}{L}\right)^{1/3} \left(\frac{\mu_b}{\mu_s}\right)^{0.14}$ Parallel plates: Nu =  $7.54 + \frac{0.03(D_h/L) \text{ Re Pr}}{1 + 0.016[(D_h/L) \text{ Re Pr}]^{2/3}}$ For fully developed turbulent flow with smooth surfaces, we have  $f = (0.790 \ln \text{Re} - 1.64)^{-2}$   $10^4 < \text{Re} < 10^6$ Nu =  $0.125f \text{ Re Pr}^{1/3}$ 

$$N_{\rm u} = 0.023 \ {\rm Re}^{0.8} \ {\rm Pr}^{1/3} \qquad \begin{pmatrix} 0.7 \le {\rm Pr} \le 160 \\ {\rm Re} > 10,000 \end{pmatrix}$$

Nu = 0.023 Re<sup>0.8</sup> Pr<sup>n</sup> with n = 0.4 for *heating* and 0.3 for *cooling* of fluid

$$Nu = \frac{(f/8)(\text{Re} - 1000) \text{ Pr}}{1 + 12.7(f/8)^{0.5} (\text{Pr}^{2/3} - 1)} \left( \begin{array}{c} 0.5 \le \text{Pr} \le 2000\\ 3 \times 10^3 < \text{Re} < 5 \times 10^6 \end{array} \right)$$