Ph.D. Qualifying Examination
Mathematics
Spring 2021

Notes:
- Time allowed: 2.5 hours
- Closed book/Closed notes (one 8.5×11" sheet of formulas is allowed)
- State your assumptions, methods, and procedures. Show all work.
- Calculators are allowed
- Laptops, cell phones and other electronic devices are not allowed

1a (12.5 points). Integrate

$$\int \frac{dx}{\sqrt{1+e^{2x}}}$$

(hint: variable substitution?)

1b (12.5 points). Find derivative $f'(x)$:

$$f(x)=\frac{x t^2}{t+x^2}$$

2 (25 points). Invert the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

3 (25 points). Solve the differential equation below for $y(t)$

$$\ddot{y}(t) + 2\dot{y}(t) + y(t) = 0$$

where

$$y(0) = 1, \dot{y}(0) = 0$$

4 (25 points). Calculate

$$\lim_{x \to 2} \sin(\pi x) \sqrt{\left| \frac{x+2}{x-2} \right|}$$