

# Ph.D. Qualifying Examination

## *Mathematics*

Fall 2015

### Logistics of exam:

- Exam duration: 2 hours
- Closed book / Closed Notes (one 8.5 x 11.00 in. sheet of formulas is allowed)
- 4 problems
- Calculators allowed (no cell phones, tablets, laptops, etc.)

Problem 1:

(a) Find  $\lim_{x \rightarrow \infty} \frac{\sqrt{2x^2 + 100,000\pi}}{3x - 5}$

(b) Find  $\lim_{x \rightarrow 0^+} x \ln x$ , Where  $\ln$  is the natural logarithm function

Problem 2:

Find the Maclaurin Series for the function  $y = f(x) = e^x$ . Show explicitly the first five terms then express the infinite series as a summation.

Hint: It is the same as the Taylor Series expanded around zero.

Problem 3:

Solve the following differential equations and find solutions that satisfy initial conditions when given:

$$(x^2 - 1)y' + 2xy^2 = 0, \quad y(x = 0) = 1$$

Problem 4:

a) Find the derivative of the following function

$$\frac{x}{(1-x)^2(1+x)^3}$$

b) Find the following indefinite integral

$$\int \frac{dx}{\sqrt{2-5x}}$$