Ph.D. Qualifying Examination

Engineering Mathematics

Fall 2016

Logistics Notes:

- Time allowed: 2 hours
- Closed book and closed notes; one sheet (8.50×11.00 in, 2-sided) of formulas is allowed
- 4 problems
- Calculators are allowed
- Laptops, cell phones, and similar electronic devices are not allowed

Problem 1.

Compute the first three non-zero terms in the Taylor series of $f(x) = e^x \sin x$ about $x = 0$. Use both $f(x)$ and the Taylor series approximation to compute the area under the curve on the interval $x \in [0, 1]$ and compare the results.

Problem 2.

a) Solve the following boundary value problem.

$$\frac{d^2y}{dx^2} + y = 0 \quad \text{with} \quad y(0) = y(\pi) = 0$$

b) Solve the following initial value problem.

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = A\cos\omega t \quad \text{with} \quad x(0) = a, \frac{dx}{dt}(0) = 0$$

Explain as much as you can about how the system behaves for varying values of ω.

Problem 3.

a) Find the derivative of the following function $y(x)$.

$$y(x) = \frac{e^x \cos x}{x^2 - x \sin x}$$

b) Find the indefinite integral of the following function $f(x)$.

$$f(x) = x^2 e^{3x}$$

(Hint: use integration by parts.)

Problem 4.

Compute the line integral

$$\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$$

where $\vec{\mathbf{F}} = 2xy\,\hat{\mathbf{i}} + (x^2 - 1)\,\hat{\mathbf{j}}$ and C is the spiral $r = 2\theta$, with $\theta \in (0, 5\pi/2)$.

(Hint: use polar coordinates to express $d\vec{\mathbf{r}}$.)