Ph.D. Qualifying Examination

Engineering Mathematics

Fall 2016

Logistics Notes:

- Time allowed: 2 hours
- Closed book and closed notes; one sheet (8.50 × 11.00 in, 2-sided) of formulas is allowed
- 4 problems
- Calculators are allowed
- Laptops, cell phones, and similar electronic devices are not allowed
Problem 1.

Compute the first three non-zero terms in the Taylor series of $f(x) = e^x \sin x$ about $x = 0$. Use both $f(x)$ and the Taylor series approximation to compute the area under the curve on the interval $x \in [0, 1]$ and compare the results.
Problem 2.

a) Solve the following boundary value problem.

\[ \frac{d^2y}{dx^2} + y = 0 \quad \text{with} \quad y(0) = y(\pi) = 0 \]

b) Solve the following initial value problem.

\[ m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = A \cos \omega t \quad \text{with} \quad x(0) = a, \frac{dx}{dt}(0) = 0 \]

Explain as much as you can about how the system behaves for varying values of \( \omega \).
Problem 3.

a) Find the derivative of the following function \( y(x) \).

\[
y(x) = \frac{e^x \cos x}{x^2 - x \sin x}
\]

b) Find the indefinite integral of the following function \( f(x) \).

\[
f(x) = x^2 e^{3x}
\]

(Hint: use integration by parts.)
Problem 4.

Compute the line integral
\[ \int_C \vec{F} \cdot d\vec{r} \]
where \( \vec{F} = 2xy \hat{i} + (x^2 - 1) \hat{j} \) and \( C \) is the spiral \( r = 2\theta \), with \( \theta \in (0, 5\pi/2) \).

(Hint: use polar coordinates to express \( d\vec{r} \).)