

Ph.D. Qualifying Examination Engineering Mathematics

July 2023

Logistics Notes

- Time allowed: 2.5 hours
- Closed book and closed notes; one sheet (8.5×11 in, 2-sided) of formulas is allowed
- Four problems
- Calculators are allowed
- Laptops, cell phones, and similar electronic devices with Internet access are not allowed

Show your work, including intermediate steps. State your assumptions clearly. Use as many sheets of paper as necessary to present each solution.

Problem 1 (25 points)

a) Evaluate

$$\int \frac{x^5 - x}{x^8 + 1} dx$$

b) Evaluate

$$\int \left(1 - \frac{2}{x}\right)^2 e^x dx$$

Problem 2 (25 points) Show the bound $f(x) \geq g(x)$ holds for all $x \in \mathbb{R}^2$ where

$$f(x) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \text{and} \quad g(x) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Problem 3 (25 points) Evaluate

$$\lim_{x \rightarrow \infty} \left(\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \cdots + \frac{2n-1}{2^n} \right)$$

Problem 4 (25 points) Solve the following differential equation

$$t\dot{x}(t) + x(t) = \cos(t)$$

with initial condition $x(0) = 0$.