# Ph.D. Qualifying Examination
# Engineering Mathematics

July 2024

**Logistics Notes**

- Time allowed: 2.5 hours
- Closed book and closed notes; one sheet (8:5 × 11 in, 2-sided) of formulas is allowed
- Four problems
- Calculators are allowed
- Laptops, cell phones, and similar electronic devices with Internet access are not allowed

Show your work, including intermediate steps. State your assumptions clearly. Use as many sheets of paper as necessary to present each solution.

**Problem 1 (25 points)**
a) Evaluate

$$\int x \ln \left| 1 + \frac{1}{x} \right| dx$$

b) Evaluate

$$\int \frac{\sin^3 x}{\sqrt{\cos x}} dx$$

**Problem 2 (25 points)** Let $g(t) = f(at)$ where the Fourier transform $F(\omega)$ of $f(t)$ is known. Find the Fourier transform

$$G(\omega) = \int_{-\infty}^{\infty} g(t) e^{\iota \omega t} dt$$

where $\iota = \sqrt{-1}$. What is the Fourier transform $H(\omega)$ of $h(t) = f(-t)$?

**Problem 3 (25 points)** Evaluate

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n+3)}$$

**Problem 4 (25 points)** Find the eigenvalues and eigenvectors of the matrix

$$M(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

for any $\theta \in [0, 2\pi)$