Ph.D. Qualifying Examination

Engineering Mathematics

Fall 2017

Logistics Notes:

• Time allowed: 2 hours
• Closed book and closed notes; one sheet (8.50 × 11.00 in, 2-sided) of formulas is allowed
• 4 problems
• Calculators are allowed
• Laptops, cell phones, and similar electronic devices are not allowed
Problem 1.

Compute the first three non-zero terms in the Taylor series of \( f(x) = e^{2x} \cos x \) about \( x = \pi/2 \). Use both \( f(x) \) and the Taylor series approximation to compute the area under the curve on the interval \( x \in [1, 2] \) and compare the results.
Problem 2.

a) Find a particular solution of the differential equation

\[ \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} - 4y = 2 \sin x \]

b) Solve the following initial value problem, where all constants are strictly positive.

\[ m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = A \cos \omega t \quad \text{with} \quad x(0) = a, \frac{dx}{dt}(0) = 0 \]

Explain as much as you can about how the system behaves for varying values of \( c \).
Problem 3.

a) Find the derivative of the following function \( y(x) \).

\[ y(x) = \frac{e^x \cos x}{x^2 + \sin x + 1} \]

b) Find the indefinite integral of the following function \( f(x) \).

\[ f(x) = x^3 e^{2x} \]

(Hint: use integration by parts.)
Problem 4.

Find the circulation of the field
\[ \vec{F} = (x - y) \hat{i} + x \hat{j} \]
around the circle
\[ \vec{R} = \cos t \hat{i} + \sin t \hat{j}, \quad t \in (0, 2\pi) \]
That is, compute the line integral
\[ \int \vec{F} \cdot d\vec{R} \]
(Hint: use polar coordinates to express $d\vec{R}$.)