Ph.D. Qualifying Examination

Engineering Mathematics

Fall 2017

Logistics Notes:

- Time allowed: 2 hours
- Closed book and closed notes; one sheet (8.50×11.00 in, 2-sided) of formulas is allowed
- 4 problems
- Calculators are allowed
- Laptops, cell phones, and similar electronic devices are not allowed

Problem 1.

Compute the first three non-zero terms in the Taylor series of $f(x) = e^{2x}\cos x$ about $x = \pi/2$. Use both $f(x)$ and the Taylor series approximation to compute the area under the curve on the interval $x \in [1, 2]$ and compare the results.

Problem 2.

a) Find a particular solution of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 2\sin x$$

b) Solve the following initial value problem, where all constants are strictly positive.

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = A\cos\omega t \quad \text{with} \quad x(0) = a,\ \frac{dx}{dt}(0) = 0$$

Explain as much as you can about how the system behaves for varying values of c.

Problem 3.

a) Find the derivative of the following function $y(x)$.

$$y(x) = \frac{e^x \cos x}{x^2 + \sin x + 1}$$

b) Find the indefinite integral of the following function $f(x)$.

$$f(x) = x^3 e^{2x}$$

(Hint: use integration by parts.)

Problem 4.

Find the circulation of the field

$$\vec{\mathbf{F}} = (x - y)\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}}$$

around the circle

$$\vec{\mathbf{R}} = \cos t\,\hat{\mathbf{i}} + \sin t\,\hat{\mathbf{j}}, \qquad t \in (0, 2\pi)$$

That is, compute the line integral

$$\int \vec{\mathbf{F}} \cdot d\vec{\mathbf{R}}$$

(Hint: use polar coordinates to express $d\vec{\mathbf{R}}$.)