# Ph.D. Qualifying Examination

Mathematics

Spring 2016

Logistics of exam:

- **Exam duration: 2 hours**
- **Closed book / Closed Notes (one 8.5 x 11.00 in. sheet of formulas is allowed)**
- **4 problems**
- **Calculators allowed (no cell phones, tablets, laptops, etc.)**

Problem 1: for the following geometries, calculate how much the inscribed triangle (figure a), inscribed square (figure b) and inscribed hexagon (figure c) approximate the area of the circle they are in. Calculate this area ratio between the two as a percentage.
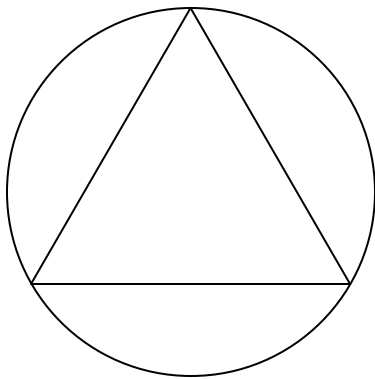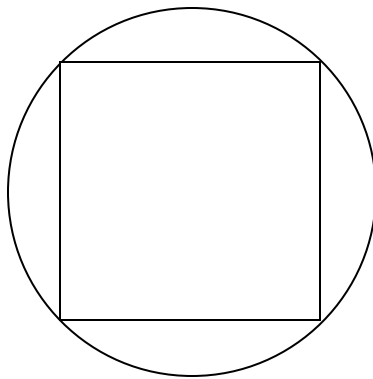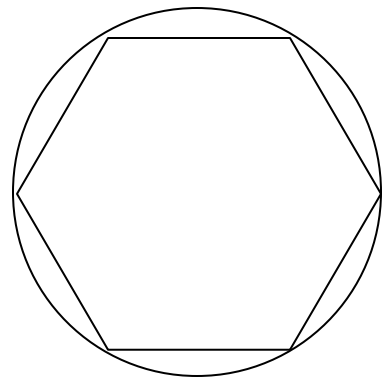
(a)   (b)   (c)

Problem 2:

(a) Find $\dfrac{dy}{dx}$ if $\sin(x + y) = y^2 \cos x$

(b) Find the equation of the tangent to the circle $x^2 + y^2 = 25$ at the point (3,4).

Problem 3:

Solve the following differential equation

$$y' - y = 2x - 3$$

Hint: use substitution such as $z = ay + bx + const$

Problem 4:

Find the derivative of the following function
$$y = \frac{\sin x - x \cos x}{\cos x + x \sin x}$$

b) Find the following indefinite integral
$$\int \frac{e^{3x} + 1}{e^x + 1} dx$$