Ph.D. Qualifying Examination
Engineering Mathematics
2022

Logistics Notes

- Time allowed: 2.5 hours
- Closed book and closed notes; one sheet (8.5×11 in, 2-sided) of formulas is allowed
- Four problems
- Calculators are allowed
- Laptops, cell phones, and similar electronic devices with Internet access are not allowed

Show your work, including intermediate steps. State your assumptions clearly. Use as many sheets of paper as necessary to present each solution.

Problem 1 (25 points): Solve the differential equation below for $x(t)$

$$\dot{x}(t) = 2\sqrt{x(t)+1}$$

where $x(0) = 0$.

Problem 2 (25 points): What is the maximum value of the function below for $x^\intercal x = x_1^2 + x_2^2 = 1$

$$f(x) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^\intercal \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ for } x^\intercal x = 1$$

HINT: Consider the eigenvalues of the matrix.

Problem 3 (25 points): Evaluate

$$\lim_{n\to\infty}\left(\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n\cdot(n+1)}\right)$$

Problem 3 (25 points): Find

$$\int \frac{dx}{x\,(x+1)\,(x^2+x+1)}$$