Ph.D. Qualifying Examination
Mathematics
Spring 2019

Notes:

- Time allowed: 2.5 hours
- Closed book/Closed notes (one 8.5 x 11" sheet of formulas is allowed).
- State your assumptions, methods, and procedures. Show all work.
- Calculators are allowed.
- Laptops, cell phones and other electronical devices are not allowed.

1a (12.5 points). Find the integral

$$\int_1^2 x \ln^2 x \, dx$$

1b (12.5 points) Find the derivative dy/dx

$$y = \frac{x^6 + x^3 - 2}{\sqrt{1 - x^3}}$$

2 (25 points). Find a general solution of the differential equation

$$y''' - 7 y'' + 15 y' - 9 y = e^x (8 x - 12)$$

3 (25 points) Invert the matrix

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$$

4a (12.5 points) Evaluate the limit

$$\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$$

4b (12.5 points) Find the sum of the series

$$\sum_{n=2}^{\infty} \frac{6}{36n^2 + 12n - 35}$$