Ph.D. Qualifying Examination

Engineering Mathematics

2020

Logistics Notes:

- Time allowed: 2.5 hours

- Closed book and closed notes; one sheet (8.5 $\times$ 11 in, 2-sided) of formulas is allowed

- 4 problems

- Calculators are allowed

- Laptops, cell phones, and similar electronic devices with Internet access are not allowed

Show your work, including intermediate steps. State your assumptions clearly. Use as many sheets of paper as necessary to present each solution.

Problem 1a (12.5 points). Find

$$\lim_{x \to \infty} \ln\left(1+2^x\right)\ln\left(1+\frac{3}{x}\right)$$

Problem 1b (12.5 points). Find

$$\lim_{x \to 0} \frac{\sinh^2 x}{\ln\left(\cosh 3x\right)}$$

Problem 2 (25 points).

Solve $\quad y''+6\,y'+13\,y=8\,e^{-x}\quad$ if at $x=0$, $y=2/3$, $y'=2$

Problem 3a (12.5 points). Compute

$$\frac{d}{dx}\sin\left(\arcsin\alpha x + \arccos\alpha x\right)$$

Problem 3b (12.5 points). Compute

$$\int x\frac{dx}{x^4 - 2x^2 - 1}$$

Problem 4 (25 points) Invert the matrix

$$\begin{pmatrix} -1 & 0 & 2 \\ 3 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix}$$